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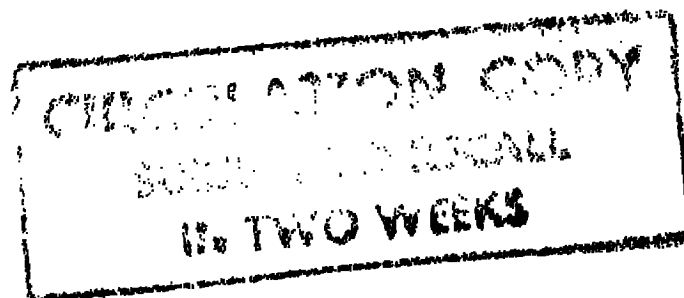
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**THERMAL EFFECTS IN DIMENSIONAL METROLOGY**



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## THERMAL EFFECTS IN DIMENSIONAL METROLOGY<sup>1</sup>

Wm. Brewer,<sup>2</sup> James B. Bryan,<sup>3</sup> Eldon R. McClure,<sup>4</sup> and J. W. Pearson<sup>5</sup>

### ABSTRACT

A Lawrence Radiation Laboratory investigation of thermal effect in dimensional metrology shows that, in the field of close tolerance work, thermal effect is the largest single source of error, large enough to make corrective action necessary if modern measurement systems and machine tools are to attain their potential accuracies. This paper is an effort to create an awareness of the thermal environment problem and to suggest some solutions. A simple, quantitative, semiexperimental method of thermal error evaluation is developed. It is shown, experimentally and theoretically, that the frequency of temperature variation is as important as the absolute limits of the temperature variation, and that the sensitivity of machine structures to thermal vibration can be minimized by selecting environmental frequencies to avoid resonant conditions. A relatively simple device to monitor the thermal environment and automatically effect error compensation is proposed.

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## I. INTRODUCTION

During a routine test of a measuring machine at the Lawrence Radiation Laboratory (LRL), it was observed that the measurements varied significantly with time. It was thought, at first, that the electronic gage used to make the measurements was the cause, but a careful check showed that the electronic drift was negligible. The measurement system was then monitored by means of a sensitive temperature pickup mounted in the air near the gage. Temperature and gage output were recorded over long time periods. The results showed a high degree of correlation between temperature variation and measurement variation.

Similar tests were conducted on many different types of measurement systems with similar results. Figure 1 is an example of such results. The significance of this effect is clear when the observed drift is compared with the working tolerance of the gage. The drift is 100 microinches and the working tolerance of the gage is only 100 microinches. In the case shown, the drift accounts for 100% of the tolerance of the gage.

As a result of this disturbing development, the Metrology Section of LRL began an investigation of thermal effect in Dimensional Metrology. As the investigation progressed, it became increasingly clear that:

1. In the field of close tolerance work, thermal effect is the greatest single source of error.
2. The usual efforts to correct for thermal error by applying expansion "correction," or by air conditioning the working area do not always solve the problem and are based on an incomplete understanding of the problem.

3. The specified accuracies of modern precision tools and gages are attainable only if the thermal environment matches the requirements of each measurement system.

It has been helpful to think of the temperature problem in terms of (1) the effects of average temperatures other than 68 degrees, (2) the effects of temperature variation about this average. The paper organization reflects this arbitrary division of the problem. There is also a discussion of ways and means of reducing thermal errors. Appendix A is a glossary of terms used to discuss thermal effects problems. Appendix B is a detailed procedure of a "drift" check and Appendix C is an outline of a method to determine the thermal frequency response of a measuring system.

## II. EFFECTS OF AVERAGE TEMPERATURES OTHER THAN 68° F

An inch is the distance between two fixed points in space. It is defined as 41,929.398742 wavelengths of the orange-red radiation of krypton-86 when propagated in vacuum. An inch does not vary with temperature. This fact is obscured because the lengths of the more common representations of the inch such as gage blocks, lead screws, and scales do vary with temperature. The lengths of most of the materials we deal with also change with temperature. In April, 1931, the International Committee of Weights and Measures Meeting in Paris agreed that when we describe the length of an object we automatically mean its length when it is at a temperature of 68 degrees. This agreement was preceded by intensive international debate and negotiation [1, 2, 3, 4, 5, 11].<sup>6</sup> This agreement means that it is not necessary to specify the measurement temperature on every drawing (no more necessary than it is to define the inch on every drawing).

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<sup>6</sup>Numbers in brackets designate Bibliography at end of paper.

If dimensions are only correct at 68 degrees, how have we been getting by all these years by measuring at warmer temperatures? The answer is that if our work is steel and our scale is steel the two expand together and the resultant errors tend to cancel. If, however, the work is another material such as aluminum, the errors are different and they don't cancel. We refer to this error as "differential expansion." We can get into the same trouble if our work is steel, but we are measuring with the "honest" inches that come from an interferometer. As a result of the discovery of the laser and the development of practical laser fringe-counting interferometers, we expect to be using more of these "honest" inches and will have to be very careful of this problem.

Knowledgeable machinists have always made differential expansion corrections. The thing that is sometimes overlooked, however, is that these corrections are not exact. Our knowledge about average coefficients of expansion is meager and we can never know the exact coefficient of each part. This inexactness we call "uncertainty of differential expansion."

This inexactness or uncertainty is zero when the average temperature is 68 degrees, and increases according to the thermal distance from 68 degrees. Its magnitude varies for different materials. We have reason to think that it is at least 5% for gage steel and on up to 25% for other materials. One metallurgist, consulted in the course of our investigation, stated that the coefficient of expansion of cast iron may vary as much as 4% between thin and thick sections. This uncertainty factor also includes the possibility of differences in expansion of a material in different directions. Differences between the actual thermal expansion and the handbook or "nominal" expansion occur because of experimental errors and because of dissimilarities between the experimental material and the material of our workpiece.

Complete studies of the errors introduced in the estimates of thermal expansion are notably absent. The data presented by Goldsmith et al. [9] show the range of disagreement among several investigators in determining the coefficient of expansion of common materials. This disagreement might be expected for some of the more exotic materials, but intuition would indicate that the knowledge of the properties of steel would be more exact. Not necessarily so, as Richard K. Kirby of the National Bureau of Standards reports<sup>7</sup>:

"The accuracy of a tabulated value of a coefficient of thermal expansion is about  $\pm 5$  percent if the heat and mechanical treatment of the steel is indicated. The precision of the coefficient (a) among many heats of steel of nominally the same chemical content is about  $\pm 3$  percent, (b) among several heat treatments of the same steel is about  $\pm 10$  percent, and (c) among samples cut from different locations in a large part of steel that has been fully annealed is about  $\pm 2$  percent (hot or cold rolling will cause a difference of about  $\pm 5$  percent)."

Corrections for uncertainty of differential expansion cannot be made. The error can be reduced by establishing more accurate nominal coefficients of thermal expansion, by improving the uniformity of coefficient of expansion from part to part through better chemical and metallurgical controls, by determining individual part and gage expansions, and by limiting the room temperature deviation from 68 degrees.

Control of uncertainty of differential expansion is the primary reason for maintaining a 68 degree average temperature. Even if we had an exact knowledge of all coefficients, the confusion and possibility of mistakes in making

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<sup>7</sup>A personal communication from Richard K. Kirby, In charge, Thermal Expansion Laboratory, Length Section, Metrology Division, U. S. National Bureau of Standards, Washington, D. C.



corrections is a second reason for maintaining 68 degrees. As our study progressed, it became necessary to establish a more exact definition of terms to facilitate rapid and clear communication. The reader should now refer to Appendix A: Glossary of Terms, definitions No. 15 through 23, which are pertinent to the discussion in this section.

Three examples are given below to illustrate the consequences of average temperatures other than 68 degrees. Possible errors are shown to be 13%, 37%, and 20% of the working tolerance. These errors do not include the effect of temperature variation which is covered in the next section. They do not include the other errors of measurement such as accuracy of standards and comparison technique. The traditional rule of ten to one allows only 10% of the working tolerance for all measurement error.

#### Example No. 1

A 10 inch long steel part with a tolerance of plus or minus a half-thousandth (500  $\mu$ in.) is measured in a C-frame comparator by comparing it to a 10 inch gage block in a room which averages 75 degrees. A handbook lists the Nominal Coefficient of Expansion (K) for the gage block as 6.5  $\mu$ in./in./deg. The K for the steel part is assumed to have the same value. The Uncertainty of Nominal Coefficient of Expansion (UNCE) for the gage block is estimated at plus or minus 5% and for the part at 10% (its exact composition is unknown). For this case, the Nominal Differential Expansion (NDE) is zero. The Uncertainty of Nominal Differential Expansion (UNDE) is, however, significant. It is the sum of the two Uncertainty of Nominal Expansion (UNE) values.

$$\begin{aligned}
 \text{NDE} &= \text{No correction necessary} &= 0 \\
 \text{UNE gage block} &= 10 \text{ in.} \times 6.5 \mu\text{in./in./deg} \times 7 \text{ deg} \times 5\% = 22 \mu\text{in.} \\
 \text{UNE part} &= 10 \text{ in.} \times 6.5 \mu\text{in./in./deg} \times 7 \text{ deg} \times 10\% = 44 \mu\text{in.} \\
 &&&\text{UNDE} = \underline{66 \mu\text{in.}}
 \end{aligned}$$

$$\frac{66}{500} \times 100 = 13\% \text{ of working tolerance}$$

### Example No. 2

A 10 inch long plastic part with a tolerance of plus or minus 0.002 inch is measured on a surface plate using an indicator stand to compare it to the readings of a Cadillac gage. The room temperature averages 75 degrees (7 degree Temperature Offset). A handbook lists the Nominal Coefficient of Expansion (K) for the gage steel assumed to be used in the Cadillac gage as 6.5  $\mu\text{in./in./deg}$ . The K value for the plastic is listed by the manufacturer as 40  $\mu\text{in./in./deg}$ . The Uncertainty of Nominal Coefficient of Expansion (UNCE) for the gage steel is estimated at 10% since we do not know the exact composition nor heat treatment. Because of past experience with plastics and a lack of any information to the contrary the UNCE for the plastic is estimated at 25%. The inspector making the measurement is thoroughly familiar with differential expansion. He computes the NDE correctly and applies it in the proper direction to the dial indicator reading which is used to transfer the Cadillac gage reading. A correction for UNDE cannot be made. Its possible value is computed below:

$$\begin{aligned}
 \text{NDE} &= \text{Corrections are made} &= 0 \\
 \text{UNE}_{\text{gage}} &= 10 \text{ in.} \times 6.5 \mu\text{in./in./deg} \times 7 \text{ deg} \times 10\% = 46 \mu\text{in.} \\
 \text{UNE}_{\text{part}} &= 10 \text{ in.} \times 40 \mu\text{in./in./deg} \times 7 \text{ deg} \times 25\% = 700 \mu\text{in.} \\
 &&&\text{UNDE} = \underline{746 \mu\text{in.}}
 \end{aligned}$$

$$\frac{746}{2000} \times 100 = 37\% \text{ of working tolerance}$$

Example No. 3

A 10 inch long aluminum part with a tolerance of plus or minus 0.001 inch is measured on a surface plate using an indicator stand to compare it to the readings of a Cadillac gage. The room temperature averages 70 degrees. (2 degree Temperature Offset.) As in the previous example, the NCE for the gage is assumed to be 6.5  $\mu$ in./in./deg. The NCE for the aluminum part is assumed to be 13.5  $\mu$ in./in./deg. The UNCE for the gage is estimated at 10% and for the aluminum part at 20%. The inspector in this case does not appreciate the magnitude of NDE, and arbitrarily decides that 70 degrees is "close enough," he does "not bother" with an NDE correction to his readings. The possible error is computed as follows:

$$\text{NDE} = 10 \text{ in. } (13.5 - 6.5) \times 2 \text{ deg} = 140 \mu\text{in.}$$

UNDE:

$$\text{UNE}_{\text{gage}} = 10 \text{ in. } \times 6.5 \times 2 \text{ deg} \times 10\% = 13 \mu\text{in.}$$

$$\text{UNE}_{\text{part}} = 10 \text{ in. } \times 13.5 \times 2 \text{ deg} \times 20\% = 52 \mu\text{in.}$$

$$\text{UNDE} = \underline{\quad\quad\quad} 65 \mu\text{in.}$$

$$\text{NDE plus UNDE} = \underline{\quad\quad\quad} 205 \mu\text{in.}$$

$$\frac{205}{1000} \times 100 = 20\% \text{ of working tolerance}$$

In the above examples we assumed that the average temperature of the gage and part were the same as the average temperature of the room. If adequate time has been allowed for the gage and part to "soak out" and reach thermal equilibrium this is a reasonable assumption. Unfortunately, this assumption does not apply to the instantaneous temperature of these components. Instead, the environment is continually varying around some mean value. The result is that differences in temperature in the various components are dynamically

induced in the system. The next section discusses the errors caused by variation in thermal environment.

### III. EFFECTS OF VARIATION IN THERMAL ENVIRONMENT

#### The Two-Element System

All length measuring apparatus can be viewed as consisting of a number of individual elements arranged to form a "C." Figure 2 shows a schematic of a C-frame comparator measuring the diameter of a short section of hollow tubing. The comparator frame and the part form two elements. If the coefficient of expansion of the comparator is exactly the same as the part, the gage head will read zero after soak-out at any uniform temperature that we might select. If we induce a change in temperature, however, the relatively thin section of the tubing will react sooner than the thick section of the comparator frame and the gage head will show a temporary deviation. The amount of the deviation will depend on the rate of change of temperature. If the rate is slow enough to allow both parts to keep up with the temperature changes, there will be a small change in gage head reading. If the rate is so fast that even the thin tubing can't respond, there will again be a small change in reading. Somewhere in between these extremes there will be a frequency of temperature change that results in a maximum change in reading. This is somewhat similar to resonance in vibration work.

To confirm our intuition on the nature of these effects the above model was further simplified to that shown in Fig. 3. Sample heat transfer calculations were made for this model and programmed on an analog computer. The

cylinder with the displacement pickup can be considered the comparator. Both cylinders are made of steel and are 4 inches long. Cylinder A is 2 inches in diameter and Cylinder B is 1/2 inch in diameter. Figure 4 shows the computer-predicted changes in length of the two cylinders as a result of a plus and minus one degree sinusoidal change in air temperature having a frequency of one cycle per hour. The thick cylinder shows less than one-third of the temperature change of the thin cylinder and its temperature lags the thin cylinder by 3 or 4 minutes. The dotted line in Fig. 4 shows the predicted gage head reading which is the same as the instantaneous difference in the lengths of the two cylinders. We call this the "Thermal Drift" (definition 24) of the system. The effect of varying the "Thermal Vibration" frequency is plotted in Fig. 5. As our intuition predicted, the drift is small for very high or low frequencies and reaches a maximum amplitude at a point in between which we call resonance. Figure 5 is called the "Frequency Response" (definition 14) of the system. In this case resonance occurs at 1/2 cycle per hour and has a value of 15  $\mu$ in. This error would occur even if the Time Average Temperature of the environment and all mechanical elements was 68° exactly.

Fifteen  $\mu$ in. may appear to be a negligible magnitude, but real measuring machines and machine tools, of course, don't have uniform coefficients throughout, and real workpieces can have quite different coefficients. This makes the effects of temperature variation much worse. If the part were Lucite, for example, these responses would be much more severe. Real systems generally have much longer overall lengths and more severe differences in mass between elements. Magnitudes of 150  $\mu$ in. per degree at resonant frequency are not unusual. Rather than mass alone, the more significant factor is the ratio of cubic inches of volume to the square inches of surface exposed to the air. This

ratio is proportional to the "Time Constant" (definition 13) of the element. The time constant is discussed in the following heat transfer calculations, which support the above results. A complete understanding of these equations is not necessary, because the important conclusions have been presented above.

We have used a gage frame to illustrate the effect of temperature variation, but it should be emphasized that the same thing happens to machine tool frames. Deflection due to temperature variations is common to all machine structures whether they be measuring machines or machine tools.

#### Calculations for Frequency Response of Two-Element System Model

To simplify the calculations, the following assumptions have been made:

a. The bodies always have uniform temperatures, i. e., there is no resistance to heat transmission between the parts of the body and any heat added simply raises the temperature at all points uniformly and instantaneously.

b. The temperature of the air surrounding the bodies is uniform at  $T_e$ .<sup>8</sup>

c. All heat transmission to and from the body is governed by Newton's law of cooling:

$$q = hA(T - T_e)$$

where A is the surface area of the body in  $\text{ft}^2$ , h is a film coefficient defining the ability of heat to pass from air to the body, in  $\text{Btu/hr-ft}^2 \cdot ^\circ\text{F}$ , and q is the rate of heat in-flux,  $\text{Btu/hr}$ .

d. The heat stored in the body is proportional to the thermal capacitance of the body or that

$$q_s = C_p \rho V dT/dt$$

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<sup>8</sup>Keeping in mind that radiative and conductive environments can exist, we shall limit the following discussion to the effect of a convective environment on the measurement process and the resulting error.

where  $q_s$  is the rate of heat storage,  $C_p$  is the specific heat in Btu/lb °F,  $V$  is the volume of the body in ft<sup>3</sup>,  $\rho$  is the density, pounds-mass per ft<sup>3</sup>, and  $dT/dt$  is the rate of change of body temperature with time. Since the heat influx must equal the heat stored in any interval of time

$$q = hA(T - T_e) = q_s = CpV\rho \, dT/dt,$$

solving for  $T_e$  yields the differential equation describing the system:

$$T + \tau \, dT/dt = T_e \tag{1}$$

where

$$\tau = \frac{CpV\rho}{hA} = \frac{\text{Thermal capacitance}}{\text{Thermal resistance}}$$

Because of assumption (a), equation (1) is only approximately correct. However, for metallic objects the thermal conductivity is high enough to make the approximation reasonable.

Equation (1) is well known in the literature on the analysis of linear systems [10] in which all elements it represents are called "time-constant elements" and  $\tau$  is called "time constant" of the element.

Given that  $T_e$  varies sinusoidally around some mean  $T_{e0}$ , i. e.,

$$T_e = (T_{e\max} - T_{e0}) \sin \omega t \tag{2}$$

where  $\omega$  is the frequency of oscillation in radians per unit time, the solution to equation (1) gives:

$$T = (T_{e\max} - T_{e0}) \sin (\omega t + \phi) (1 + \omega^2 \tau^2)^{-1/2} \tag{3}$$

where the phase lag angle  $\phi$  is given by

$$\phi = \tan^{-1} \omega \tau$$

When this solution is applied to the two-element system we obtain the relationships between the temperatures of the two elements and the environment temperature as discussed above.

#### Drift Check

Measurement of the drift in a measurement system is called a "drift check" (definition 26). To make a drift check it is merely necessary to indicate from the comparator to the master, or part as the case may be, and record the relative motion between the elements under the normal conditions of the measurement process. This procedure has been made possible by the development of high sensitivity, drift-free displacement transducers and recorders. The electronics drift check (definition 25) provides a simple means of proving the stability of these devices. Our experience indicates that "electronics drifts" of less than  $3 \mu\text{in.}$  per day for  $\pm 3^\circ$  environments can be expected. Details of drift-check procedures are given in Appendix B.

#### Predicting the Effects of Temperature Variation

The mathematical approach given above allows us to make quantitative observations about the effects of thermal vibration in systems for which we know all the time constants. The drift check provides us with a practical means of error evaluation on real systems in a given environment regardless of their complexity. Neither of these approaches can provide us with a means of determining how large the errors will be in real systems before they are installed in a given environment. Such information is necessary if rational design decisions are to be made. Therefore, our investigation included a study of the means to experimentally determine the dynamic response of measurement systems and to find ways to predict, from this information, what the drift will be for any system in any environment.



A study of the literature [10] on the analysis of linear systems shows that it is possible to conduct "step input" tests, the results of which provide a means of approximating the effects of any kind of change. To determine the feasibility of applying this procedure to a real measurement process, a series of experiments was conducted in an LRL inspection shop. In these experiments the apparatus consisted of a 15 inch Sheffield rotary contour gage measuring a hollow steel hemispherical part, as shown in Fig. 6.

The Sheffield gage chosen was particularly suited to these experiments because it was located in a room that had a particularly good air-conditioning system. Room-air temperatures in the vicinity of the gage responded to a one degree change in the set point of the air-conditioning controller within several minutes.

Linearity of the system was established in three experiments which consisted of suddenly raising the set point of the controller 1 degree in the first, lowering it 1 degree in the second, and raising it 2 degrees in the third. Air temperature at a point just above the part was recorded using a thermister magnetically held in contact with a 10 inch piece of 0.010 inch shim stock. Resulting drift was recorded by the equipment on the gage.

The results from this series of experiments were compared. All three drift curves showed the effects of a high degree of linearity. They differed only in magnitude and this disagreement was less than 10%.

Subsequently, an arbitrary temperature fluctuation was imposed on the room by driving the controller set point with a motor-driven cam mechanism. Temperature and drift were recorded as before.

The recorded drift and the corresponding recorded temperature changes for the one-degree step input change experiment were used as shown in Appendix C to compute a theoretical drift from the forced-drift temperature data. Figure

7 shows the step input temperature change and drift profiles and Fig. 8 the recorded and computed drift. Considering the fact that the experiment continued over a period of about 6 weeks, we think the results fully justify the applicability of this type of system testing.

These results encouraged us to use the computation method to calculate the frequency response of the system with the results shown in Fig. 9. Comparing these data with those in Fig. 5, we see the typical pattern as well as the distorting effect of additional time-constant elements.

The next question to be answered is: "Can data obtained on a system in one environment be applied to a similar system in another environment?" If the answer is yes, this means that a gage manufacturer, by conducting these simple step input tests in his laboratory, can provide information that will allow the customer to decide whether or not his environment is suitable for the gage.

To answer the question, we conducted a normal drift check (shown in Fig. 1) on a second 15 inch Sheffield rotary contour gage located in a different room with a different environmental control system. We used the recorded temperature variation from this system and the frequency-response information obtained from the first system to compute a predicted drift. The results of this experiment are shown in Fig. 10. The correspondence between the computed and actual drift is impressive, and though the method used must be tried for a large variety of cases before we know how general it is, we feel confident that the affirmative answer has been obtained.

Application of this method requires a high quality, temperature controlled room large enough to house the completed machine. The room must have the ability to hold a given temperature to a tolerance that makes the step input change

significant. The time required for the room to stabilize at the new temperature should be a small fraction of the soak-out time of the machine.

It would be more convenient if we could arrive at a method for making these predictions by analyzing the results of an ordinary drift check taken in an ordinary environment. At the present time, achievement of this goal appears to be difficult, but possible. The difficulty is the dependence of the micro-inch drift on the frequency of temperature variation as well as its amplitude. Work on the solution of this problem is now underway.

### The Three-Element System

All length measurement systems can be discussed in terms of three elements, a part, a master, and a comparator used to compare the part with the master. The master is sometimes obscured because it is combined with the comparator, as in a micrometer. In a micrometer, the screw is the master and the rest of the device is the comparator. If the master and the comparator are not combined, a time element is introduced into the measurement process because the comparator, such as a pair of calipers, cannot be mastered, or set, at the same time it is used to indicate on a part. This time lapse is the difference between a two-element system and a three-element system. We have already seen that the different response of part and comparator in a two-element system causes a drift error. A similar error will occur between master and comparator. This means that, in a three-element system, there is a master-comparator drift error that must also be considered to get the maximum temperature variation error. The two drift curves, between part and comparator and between master and comparator, can be used to approximate this temperature variation error in a three-element system for any mastering time cycle.

If the mastering time is zero or insignificantly small, then the comparator is slaved to the master and the temperature variation error is the drift between part and master over a representative time period. This is equivalent to the two-element system discussed previously. The representative time period is usually a working day, but may be shorter or longer depending on environment control and work habits. It should be long enough to cover the entire temperature cycle of each measurement situation. The drift of part and master cannot usually be compared directly but can be compared indirectly by comparing the part-comparator drift curve with the master-comparator drift curve. The maximum excursion of the two curves for the same temperature phase and amplitude over the representative time period will provide the maximum part-to-master drift error. This error is an approximation because the temperature conditions of the two drift checks will never be identical.

Figure 11 shows part and master drift curves. For simplicity they are made sinusoidal and in phase. The curves show absolute drift in length from an average temperature of  $68^{\circ}$  at which point they are equal length. It can be seen that measuring the part at any time other than when the part and master curves are at the same point will result in an error reading. The maximum error will occur when the part is measured at the point of maximum difference. The part is, of course, measured with the comparator, but with zero mastering time, the comparator length is held to the master length at measuring time.

If the comparator cannot be used to indicate on a part at the same time it is mastered, then the drift of the comparator with respect to part and master becomes an additional source of error. It can be shown that the maximum possible temperature variation error for a finite mastering cycle time will not be greater than the already determined maximum error from part-to-master drift

unless either the total part-to-comparator drift or the total master-to-comparator drift during the mastering cycle time is more than twice the maximum part-to-master drift error.

Figure 12 shows the absolute drifts for part, master, and comparator. The phase lag shown is typical for the varying responses to temperature variation. In the example shown, the maximum part-to-master drift (B) is the maximum temperature variation error for zero mastering cycle time. Mastering the comparator at the time shown will displace the comparator drift curve. It can be seen that the subsequent maximum temperature variation error will now vary with the mastering cycle time between mastering and indicating. The true error for mastering cycle time number 1 is  $X_1$ . This can be approximated by measuring the peak-to-valley drift of part-to-comparator or master-to-comparator, whichever is greater, and subtracting the part-to-master drift. In the example shown, the part-to-comparator drift, A, is greater so that  $A - B$  conservatively approximates the true error  $X_1$ . The same is true for mastering cycle time number 2. In each case,  $A - B$  is greater than B so  $A - B$  is the maximum temperature variation error for those conditions. If B were greater than  $A - B$ , B would remain the maximum temperature variation error because indicating could be done at any time during the mastering cycle including shortly after mastering.

Figure 13 shows what happens when the drift rate of the comparator is between that of the master and of the part. In this example the part-to-master drift error is B and the master-comparator drift is A. Figure 13 shows that  $A - B$  can never be greater than B, so part-to-master drift remains the maximum temperature variation error regardless of mastering cycle time. This condition of a comparator drift rate between that of master and part drift rates becomes apparent when the two drift curves, part-comparator and master-comparator,

are compared for part-to-master drift. If the two drift curves when aligned for temperature phase are out of phase, the comparator drift must be between master and part and the maximum temperature variation error becomes that of maximum excursion between the two curves over the representative time period.

The general case for maximum temperature variation error approximation can be stated as:

For zero or insignificantly small mastering cycle time

$$\text{TVE} = \left[ \begin{array}{l} \text{Maximum excursion of part-comparator drift} \\ \text{from master comparator drift over represen-} \\ \text{tative time period.} \\ \text{(part-to-master drift error)} \end{array} \right]$$

For a significant mastering cycle time, take either the maximum part-comparator drift or the maximum master-comparator drift, whichever is greater, for the mastering cycle time period chosen and subtract the part-to-master drift error previously chosen.

$$\text{TVE} = \left[ \begin{array}{l} \text{Master-comparator drift or} \\ \text{part-comparator drift, which-} \\ \text{ever is greater, for mastering} \\ \text{cycle time period chosen} \\ \text{A} \end{array} \right] - \left[ \begin{array}{l} \text{Part-to-master} \\ \text{drift} \\ \text{B} \end{array} \right]$$

If the result of A - B is greater than B then that result is the maximum temperature variation error for the chosen mastering cycle time. If B is greater, then the part-to-master drift error remains the maximum temperature variation error. A will not be more than 2B, and will therefore not add to the part-to-master drift error if the comparator is made to have a drift rate between that of the part and the master regardless of mastering cycle time. Also, A will not be more than 2B if the mastering cycle time is kept short enough to prevent the peak-to-valley drift of the comparator from either part or master

comparator combination while simulating the actual conditions of the measurement process. Both the master-comparator and the part-comparator drift checks are analyzed to determine the value of the maximum drift of each occurring within a time period equal to the mastering cycle. Both the master-comparator and part-comparator drift checks are then analyzed to determine the maximum excursion of the drift curves that occurs within a "representative" time period. This "representative" time period is somewhat difficult to define. It should be long enough to reveal the full pattern of temperature variation. In most cases, a period of 24 hours is sufficient. The results of the drift check analyses are then substituted into the following expression for Temperature Variation Error (TVE):

For zero or small mastering cycle time.

$$TVE = \left[ \begin{array}{l} \text{Maximum excursion of part-comparator and master-} \\ \text{comparator drift curves when curves are aligned for} \\ \text{in-phase temperature conditions over representative} \\ \text{time period.} \\ \text{(Part-to-master drift)} \end{array} \right]$$

For significant mastering cycle times.

$$TVE = \left[ \begin{array}{l} \text{Master-comparator drift or} \\ \text{part-comparator drift, which-} \\ \text{ever is greater, for chosen} \\ \text{mastering cycle time} \end{array} \right] - \left[ \begin{array}{l} \text{Part-to-master} \\ \text{drift error as above} \end{array} \right]$$

Use whichever of above TVE is greater.

The temperature variation error is combined with the Nominal Differential Expansion (NDE) and the Uncertainty of Nominal Differential Expansion (UNDE) to obtain the Thermal Error Index (TEI). The plan consists of:

1. Computing the Nominal Differential Expansion (NDE).
2. Computing the Uncertainty of Nominal Differential Expansion (UNDE).
3. Determining the Thermal Variation Error (TVE) by evaluation of drift check data.

4. Summing the absolute values obtained in 1, 2, and 3.
5. If NDE corrections are made, NDE is not included in the above sum.

The time-honored rule of 10/1 suggests that the total measuring error be limited to 10% of the working tolerance. We have found, however, that the error due to temperature is, in most cases, so large that in order to stay within economic reality we must plan on giving up the full 10% and more for temperature alone.

The following example shows how the Evaluation Plan is used in practice. This example is the same as the one used in Section II but now includes the effects of temperature variation:

A 10 inch long steel part with a tolerance of plus or minus a half-thousandth ( $500 \mu\text{in.}$ ) is measured in a C-frame comparator by comparing it to a 10 inch gage block in a room which averages 75 degrees. A handbook lists the Nominal Coefficient of Expansion (K) for the gage block as  $6.5 \mu\text{in./in./deg.}$  The K for the steel part is assumed to have the same value. The Uncertainty of Nominal Coefficient of Expansion (UNCE) for the gage block is estimated at plus or minus 5% and for the part at 10% (its exact composition is unknown). For this case, the Nominal Differential Expansion (NDE) is zero. The Uncertainty of Nominal Differential Expansion (UNDE) is, however, significant. It is the sum of the two Uncertainty of Nominal Expansion (UNE) values.

A 24-hour drift check between the comparator and master gage block shows a  $300 \mu\text{in.}$  peak-to-valley drift. The comparator is normally remastered every 2 hours. Interpreting the drift checks for maximum drift in 2 hours gives a maximum value of  $30 \mu\text{in.}$  Because the part has fewer cubic inches of volume per square inch of surface than the gage block (its time constant is smaller) the time constant



mismatch to the relatively heavy comparator frame is worsened. The part-comparator drift is found to be 350  $\mu$ in. in 24 hours. Substituting into the Evaluation Plan for the above conditions yields the following:

$$\begin{aligned} \text{NDE} &= \text{No correction necessary} &= 0 \\ \text{UNE gage block} &= 10 \text{ in.} \times 6.5 \mu\text{in./in./deg} \times 7 \text{ deg} \times 5\% &= 22 \mu\text{in.} \\ \text{UNE part} &= 10 \text{ in.} \times 6.5 \mu\text{in./in./deg} \times 7 \text{ deg} \times 10\% &= 44 \mu\text{in.} \\ &&\text{UNDE} = 66 \mu\text{in.} \end{aligned}$$

$$\text{TVE} = \left[ \begin{array}{c} \text{A} \\ \text{Maximum comparator drift from} \\ \text{part or master over 24 hours} \\ = 30 \mu\text{in.} \end{array} \right] - \left[ \begin{array}{c} \text{B} \\ \text{Part-to-master} \\ \text{drift} = 50 \mu\text{in.} \end{array} \right]$$

Use A - B or B, whichever is greater

$$(30 - 50) < 50$$

Therefore:

$$\begin{aligned} \text{TVE} &= 50 \\ \text{TEI} &= \frac{50}{116} \end{aligned}$$

$$\frac{116}{500} \times 100 = 23\% \text{ of the working tolerance}$$

The above example shows a thermal error index of more than 10% and corrective action is indicated. If, however, the tolerances increased or we decided to accept a higher percentage thermal index the situation would return to normal. A "bad" environment would suddenly become a "good" environment which does not justify the cost of any improvements. The Evaluation Plan is a way of estimating the temperature problem for each shop, each machine, and each job. It can tell us whether or not we need to improve our temperature control and by how much. The plan provides concrete economic justification for investment of the large amount of money that may be necessary to control the temperature problem. It can also prevent overdesign in the situations where it has become stylish to have special temperature controlled areas. It substitutes

an orderly thinking process for emotion or arbitrarily set rules. Natural priorities are established to indicate where our improvement efforts should be made. Should we try to move closer to 68 degrees, or should we try to reduce our temperature variation? The plan not only answers these questions, it gives a positive response to any improvements that may be made.

In spite of the advantages of the plan, some objections have been raised. One objection is that the plan pretends to be an exact procedure when obviously we are still estimating. Our answer to this objection is to agree that the plan is not perfect and not exact. It may be in error by 25% or more and still be a significant advancement over no plan at all. No plan at all means that we must depend on the opinion of experts who arbitrarily decide that this or that environment is, or is not, acceptable.

## V. METHODS FOR DECREASING THERMAL ERROR INDEX

### Average Temperature Other Than 68°

The possibilities for controlling the error resulting from average temperatures other than 68° are limited. They can be summarized in one sentence: The error can be reduced by making nominal differential expansion corrections, by establishing more accurate nominal coefficients of expansion, by improving the uniformity of coefficient of expansion from part to part through better chemical and metallurgical controls, by determining individual part expansions, and by limiting the room temperature deviation from 68 degrees.

### Temperature Variation Error

What are some of the things we can do to improve the ability of a gaging system to withstand temperature variation? Our first reaction is to make the thermal response of the master and comparator equal. This will result in zero drift between the master and comparator. Shortening the mastering cycle has

the same effect. This is a false goal, however, because we may create an increased mismatch to the part. A worthwhile goal is to make the thermal response of all three elements the same. This completely eliminates the problem but is not a practical approach because most gages are used for more than one part. The best compromise is to design the comparator drift to be about half-way between the part drift and the master drift (this is discussed in more detail in Section III). Adjustment of thermal response can be accomplished in several ways. The use of Invar is quite practical. Invar is readily obtainable at a reasonable cost and has a coefficient of only  $1 \mu\text{in./in.}$  Time constants can be controlled by use of insulation and by proper design of wall thickness.

Unfortunately, none of these solutions can be applied to the part itself. We can't insulate it; we can't change its coefficient; we can't change its wall thickness. The only thing we can do is improve the environment.

What are some of the things that can be done to improve the environment? Our first reaction is to simply reduce the temperature excursion of the whole room. This is effective, but also expensive. It may be cheaper to control the temperature excursion in a small area around the machine. The Moore Special Tool Co. of Bridgeport, Conn., uses this approach in comparing and calibrating their ultraprecise step gages to an accuracy of one part in ten million.

Another approach is the possibility of increasing the rate of cycling of the room. The frequency response diagram of the rotary contour gage (Fig. 9) shows the advantage of mismatching the environmental frequency and the resonant frequency of the gage. Because the resonant frequencies of real gaging systems are so slow (in this case 14 hours per cycle), this mismatching is best accomplished by increasing the environmental frequency. Interpreting Fig. 9 we see that a plus or minus 1 degree temperature control at 0.07 cycle per hour gives the same drift as a plus or minus 4 degree control would give at one cycle per

hour. In some cases it is possible to increase the rate of room cycling by a simple readjustment of the thermostat. The results can be quite dramatic.

High cycling rates are generally achieved by circulating large volumes of air. High volume air circulation is not too expensive and offers several advantages. A greater volume of air requires a smaller temperature difference between the inlet and outlet to maintain the same room average. This is simply a matter of removing the same number of Btu's with more pounds of air at a smaller temperature difference. Another advantage of air volume is that the increased velocity tends to scrub the whole gaging system and remove the heat that may be coming from external point sources of heat such as motors, lights, people, and radiation from the sun. Stated more exactly, the increased air velocity increases the convective heat transfer coefficient and decreases the thermal resistance between the gage and the room air, which is the thing that is being controlled. Still another advantage of high air flow is increased operator comfort. The decreased difference between inlet and outlet air temperatures means fewer cold drafts which are the real source of discomfort.

The benefits of high air flow, high cycling rates, and close containment of sensitive equipment have recently been demonstrated at LRL. A new rotary contour gage has just gone into service which is completely enclosed in a plexiglass box. Air is admitted through a plenum chamber at the top and leaves through a plenum chamber at the bottom. The circulation rate is one complete change of air every 3 seconds. The cycling rate is 25 cycles per hour. The room temperature variation is 0.7 degrees, but a 24-hour drift check shows less than 3  $\mu$ in. of drift!

The problem of standardization of room air temperature measurement is illustrated by the different values obtained on this system with three different ways of measuring. High sensitivity mercury thermometers show less than 0.05 degree

variation. The thermister recorder-controller for the enclosure shows a 0.4 degree variation and a high frequency response thermograph shows 0.7 degree.

### An Automatic Error Correcting Device

In reviewing our experiences with computing thermal drift from knowledge of system frequency response and measurement of temperature variation, Mr. J. W. Routh, of LRL, suggested that we consider the possibility of automatic error correction. Preliminary investigation of this idea has convinced us that it should be possible to design a thermal model of the system that can sense the room temperature and provide an electrical output equal to the drift. This output can be used to zero shift the coordinate system of the gage and provide direct, on line, compensation for thermal error. As it is now visualized, this device would be completely automatic once set for the specified part to be measured. The operational settings required would be nominal coefficient of expansion, time constant, and size of the part. The response of the gage would be built into the device. Some adjustment might be required for different setups that might be encountered. If the time constant of the master could be tailored to match the gage, the bulk of the thermal error could be eliminated. Error due to uncertainty of nominal differential expansion would still remain.

While this manuscript was being prepared, a report of a feasibility study by a graduate student at the University of California, Berkeley [12] concerning the practicability of such a device became available. This study was made at our request and has shown that:

- (1) A simple analog, consisting of only two time-constant elements in parallel, provides an adequate model of the system.
- (2) The main problem encountered in constructing the compensating device was in finding practical, long-time-constant elements.

This device, if realizable, will have far-reaching effects on the use and design of machine tools and measuring machines.

## VI. CONCLUSIONS

In conclusion the following generalized approach to the problem of thermal effects in dimensional metrology is suggested:

- A. Evaluate existing conditions to determine whether or not a problem exists. This is accomplished by substituting existing conditions into the Evaluation Plan. If the thermal error index is more than 10% of the part tolerance, it is likely that a problem does exist.
- B. Review the working tolerances to be sure they are economically and functionally realistic.

- C. If necessary, take corrective action to reduce the thermal error index as follows:

To reduce error resulting from average temperatures other than 68°:

1. Make corrections for nominal differential expansion.
2. Establish more accurate coefficients of expansion so as to increase the accuracy of the corrections.
3. Minimize average temperature deviations from 68° F.

To reduce error resulting from temperature variation:

1. Improve procedures for soaking out workpieces and masters so they are in thermal equilibrium with the environment.
2. Shorten the mastering cycle time if indicated by the Evaluation Plan.
3. Increase rate of air flow and improve its distribution.
4. Increase the frequency of temperature variation.

5. Decrease the amplitude of temperature variation.
6. Redesign masters and comparators so their time constants and coefficients of expansion are in better balance with those of the parts to be measured.

#### APPENDIX A: GLOSSARY OF TERMS

1. Part or Workpiece: In every length determination process, there is some physical object for which a linear dimension is to be determined. This object is called the part or workpiece.
2. Master: In the length measuring process, the unknown or desired dimension of the part is compared with a known length called the master. This length may be the wavelength of light, the length of a gage block, line standard, lead screw, etc.
3. Comparator: Any device used to perform the comparison of the part and master is called a comparator.
4. Mastering: The action of nulling a comparator with a master is called mastering.
5. Mastering Cycle Time: The time between successive masterings of the process is called the mastering cycle time of the process.
6. Measurement Process: All of the activities of which a measurement is composed is called the measurement process.
7. Measurement System: The entire apparatus used in making a measurement is called the measurement system.
8. Thermal Environment: Any physical object is exposed to various sources (and sinks) of heat energy which influence its thermal state. Taken in toto all such sources and sinks form the thermal environment of the object. In

the laboratory or shop, the thermal environment of any object can consist of all other objects with which the object is in thermal communication, i. e., by convection, conduction, and radiation. Sources and sinks commonly found in the laboratory are:

1) Convection sources and sinks:

.. Air atmosphere, including the air-conditioning system and distribution or flow of the air. The air constitutes the medium of convection heat transfer.

2) Radiant sources:

- a) Sun (if windows exist)
- b) Walls, floor, and ceiling
- c) Illuminating lights
- d) Electric motors
- e) People

3) Conductive sources are usually the most obvious, and include all objects in direct contact.

In this sense, then, an object in an air-conditioned room is in thermal communication with the air-conditioner by, usually, convection. It may also be in communication with an electric motor by convection, conduction, and radiation.

Although, in the general case, it is probable that all types of thermal communication exist between the environment and a given object, perhaps the most common environment is the one in which the only significant communication is by convection. In this case, the effect of the environment on the object can be described in terms of thermal state of the volume of air surrounding the object.



8(a) Convective environment.

When all environmental influences are convective in nature and a single temperature describes the environment, the environment is called a convective environment. The response of an object (changes in length) in such an environment can be directly correlated with the environment temperature.

8(b) Environment temperature,  $T_e$ .

The temperature by which the thermal state of a convective environment is measured is called the environment temperature.

8(c) Temperature offset.

The difference between the time average of the environment temperature and  $68^\circ\text{F}$  is called temperature offset.

$$TO = T_e - 68^\circ$$

9. Variations of Thermal Environments

9(a) Stationary environment.

When the environment is invariable in time, it is called stationary.

9(b) Periodic environment.

An environment in which every variable changes in a cyclic manner is called a periodic environment.

9(c) Aperiodic environment.

(1) Transient environment.

When the environment change is not periodic but has a well-defined pattern, such as a constant rate of increase of temperature in a convective environment, it is called a transient environment.

(2) Random environment.

When the environment changes in a random manner, it is called a random environment. Influences due to the presence of human beings or weather tend to be random.

Although all environments have some random characteristics, deliberate attempts at environmental control, e. g., by refrigerant air conditioning, tend to introduce dominant periodic characteristics. Also, in uncontrolled environments transient characteristics may be found to dominate. For example, the outside air temperature may dominate in a room which is well ventilated.

10. Standard Temperature for Length Measurements: Unless otherwise specified, the dimensions of an object given in drawings or specifications shall be for an object with a uniform temperature of 68° F (20° C). A length of an object at standard temperature is called the standard length of the object. This procedure follows the April 1931, resolution of the International Committee of Weights and Measures that the temperature of 20° C (68° F) should be universally adopted as the normal temperature of adjustment for all industrial standards of length. Also, Recommendation No. 1 of the International Organization for Standardization, issued in 1954, promulgates the standard temperature of 20° C among the 40 participating countries.

11. Temperatures of a Body

11(a) Temperature (at a point).

When discussing a body which does not have a single uniform temperature, it is necessary to refer in some manner to the distribution of temperature throughout the body. Temperature

at a point in a body is assumed to be the temperature of a very small volume of the body centered at that point. The material of which the body is composed is assumed to form a continuum.

11(b) The temperature of a body.

When the differences between the temperatures at all points in a body are negligible, the body is said to be at a uniform temperature. This temperature is then the temperature of the body.

11(c) Instantaneous average temperature of a body.

When the body is not at a uniform temperature at all points, but it is desirable to identify the thermal state of the body by a single temperature, the temperature which represents the total heat stored in the body may be used. When the body is homogeneous this temperature is the average, over the volume of the body, of all point temperatures. This is called the average temperature of the body.

11(d) Time-mean temperature of a body.

The time average of the average temperature of a body is called the time-mean temperature of the body.

12. Soak Out: One of the characteristics of a thermal system is that it has a "memory." In other words, when a complete change in environment is experienced, such as occurs when an object is transported from one room to another, there will be some period of time before the object completely "forgets" about its previous environment and exhibits a response dependent only on its current environment. The time elapsed from a change in environment until the object is influenced only by the new environment is called soak-out time. After "soak out" the object is said to be in

equilibrium with the new environment. In cases where an environment is time-variant the response of the object is also a variable in time. When the object exhibits a response dependent only on the environment it is said to be in dynamic equilibrium with its environment.

13. Time Constant of a Body: The time constant of a body is a measure of the response of the body to environmental temperature changes. It is defined as the time required for a body to achieve 63.2% of its total change after a sudden step change in the environment.
14. Frequency Response: The frequency response of a measurement system is defined as the ratio of the amplitude of the drift in microinches to the amplitude of a sinusoidal environment temperature oscillation in degrees Fahrenheit for all frequencies of temperature oscillation.
15. Thermal Expansion: The difference between the length of a body at one temperature and its length at another temperature is called the thermal expansion of the body.
16. Coefficient of Expansion:
  - 16(a) The true coefficient of expansion,  $\alpha$ , at a temperature,  $t$ , of a body is the rate of change of length of the body with respect to temperature at the given temperature divided by the length at the given temperature.

$$\alpha = \frac{1}{L} \frac{dL}{dt}$$

- 16(b) The average true coefficient of expansion of a body over the range of temperatures from 68° F to  $t$  is defined as the ratio of the fractional change of length of the body to the change in temperature.

Fractional change of length is based on the length of the body at 68° F,

$$\alpha_{68, t} = \frac{L - L_{68}}{L_{68} (t - 68)}$$

Hereinafter the term "coefficient of expansion" shall refer only to the average value over the range from 68° F to another temperature, t.

17. Nominal Coefficient of Expansion: The estimate of the coefficient of expansion of a body shall be called the nominal coefficient of expansion. To distinguish this value from the average true coefficient of expansion ( $K_{68, t}$ ) it shall be denoted by the symbol K.
18. Uncertainty of Nominal Coefficient of Expansion: The maximum possible percentage difference between the actual coefficient of expansion,  $\alpha$ , and the nominal coefficient of expansion shall be denoted by the symbol  $\delta$ , and expressed as a percentage of the true coefficient of expansion.

$$\delta = \frac{\alpha - K}{\alpha} (100)\%$$

Variations in material composition, forming processes, and heat treatment as well as inherent anisotropic properties and effects of preferred orientation cause objects of supposedly identical composition to exhibit different thermal expansion characteristics. Also, differences in experimental technique cause disagreement among thermal expansion measurements. As a result, it is difficult, solely from published information, to obtain an exact coefficient of expansion for any given object.

This value like that of K itself must be an estimate. Various methods can be used to make this estimate. For example:

- 18(a) The estimate may be based on the dispersion found among published data.

- 18(b) The estimate may be based on the dispersion found among results of actual experiments conducted on a number of like objects.

Of the two possibilities given above, (b) is the recommended procedure.

Because the effects of inaccuracy of the estimate of the uncertainty are of second order, it is considered sufficient that good judgment be used.

19. Nominal Expansion: The estimate of the expansion of an object from 68° F to its time-mean temperature at the time of the measurement shall be called the nominal expansion and it shall be determined from the following relationship.

$$NE = L(t - 68)(K)$$

20. Uncertainty of Nominal Expansion: The maximum difference between the true thermal expansion and the nominal expansion is called the uncertainty of nominal expansion. It is determined from:

$$UNE = L(t - 68) \left( \frac{\delta}{100} \right).$$

21. Differential Expansion: Differential expansion is defined as the difference between the expansion of the part from 68° F to its time-mean temperature at the time of the measurement and the expansion of the master from 68° F to its time-mean temperature at the time of the measurement.

22. Nominal Differential Expansion: The difference between the nominal expansion of the part and of the master is called the nominal differential expansion.

$$NDE = (NE)_{\text{part}} - (NE)_{\text{master}}$$

23. Uncertainty of Nominal Differential Expansion: The sum of the uncertainties of nominal expansion of the part and master is called the uncertainty of nominal differential expansion.

$$UNDE = (UNE)_{\text{part}} + (UNE)_{\text{master}}$$

24. Thermal Drift: Drift is defined as the differential movement of the part or the master and the comparator in microinches caused by time-variations in the thermal environment.
25. Electronics Drift Check: An experiment conducted to determine the drift in a displacement transducer and its associated amplifiers and recorders when it is subjected to a thermal environment similar to that being evaluated by the drift check itself. The electronics drift is the sum of the "pure" electronics drift and the effect of the environment on the sensing head, amplifier, etc. The electronics drift check is performed by blocking the transducer and observing the output over a period of time at least as long as the duration of the drift test to be performed. Blocking a transducer involves making a transducer effectively indicate on its own frame, base, or cartridge. In the case of a cartridge-type gage head, this is accomplished by mounting a small cap over the end of the cartridge so the plunger registers against the inside of the cap. Finger type gage heads can be blocked with similar devices. Care must be exercised to see that the blocking is done in a direct manner so that the influence of temperature on the blocking device is negligible.
26. Drift Check: An experiment conducted to determine the actual drift inherent in a measurement system under normal operating conditions is called a drift check. Since the usual method of monitoring the environment (see definition 28) involves the correlation of one or more temperature recordings with

drift, a drift check will usually consist of simultaneous recordings of drift and environmental temperatures. The recommended procedure for the conduct of a drift check is given in Appendix B.

27. Temperature Variation Error, TVE: An estimate of the maximum possible measurement error induced solely by deviation of the environment from average conditions is called the temperature variation error. TVE is determined from the results of two drift checks; one of the master and comparator, and the other of the part and the comparator.

For zero or small mastering cycle time.

$$\text{TVE} = \left[ \begin{array}{l} \text{Maximum excursion of part-comparator and master-} \\ \text{comparator drift curves when curves are aligned for in-} \\ \text{phase temperature conditions over representative time} \\ \text{period.} \end{array} \right] \quad (\text{Part-to-master drift})$$

For significant mastering cycle times.

$$\text{TVE} = \left[ \begin{array}{l} \text{Master-comparator drift or} \\ \text{part-comparator drift, which-} \\ \text{ever is greater, for chosen} \\ \text{mastering cycle time} \end{array} \right] - \left[ \begin{array}{l} \text{Part-to-master} \\ \text{drift error as above} \end{array} \right]$$

Use whichever of above TVE is greater.

28. Total Thermal Error: Total thermal error is defined as the maximum possible measurement error resulting from temperatures other than a uniform, constant temperature of exactly 68° F. It is, of course, desirable to determine the total thermal error induced in any measurement. However, this is usually not practical to do, and in many cases, not even possible. Therefore, an alternative procedure is outlined below.
29. Thermal Error Index: The evaluation technique proposed in this section does nothing more than estimate the maximum possible error caused by thermal environment conditions affecting a particular measurement process. It does not establish the actual magnitude of any error. It serves to remove doubt about the existence of the errors and to establish a system of rewards and penalties to processes which are combinations of techniques, some of which may be "good" and some "bad."



The Thermal Error Index shall apply only so long as conditions do not change.

The proposed plan consists of:

- (1) Computing the nominal differential expansion, NDE.

In this computation (and in the next), the temperature offset is assumed to be the average difference between 68° F and the air temperature in the vicinity of the process over the mastering cycle of the process.

- (2) Computing the uncertainty of NDE, UNDE.
- (3) Determining the thermal variation error, TVE, by means of a drift check.
- (4) Summing the absolute values obtained in 1, 2, and 3 to obtain an index related to the quality of the process, yields the temperature error index.

$$TEI = NDE + UNDE + TVE$$

- (5) If an effort is made to correct the measurement by computing the NDE, part 1 is to be deleted.

The plan penalizes a measurement process on two counts:

- (1) Existence of environment temperature offset, resulting in differential expansion.
- (2) Existence of environment variations.

The plan rewards good technique by reducing the thermal error index for:

- (1) Attempting a correction for differential expansion.
- (2) Keeping environmental variations to a minimum.

Thermal error index can be used as an administrative tool for certification of measurement processes as is discussed in the next section. It can also be used as an absolute index of acceptability of the process. For example, a good rule of thumb for establishing the acceptability of a measurement

process with respect to thermal errors is to limit the acceptable thermal error index to 10% of the working tolerance of the part.

30. Monitoring: To perpetuate the thermal error index it will be necessary to monitor the process in such a way that significant changes in operating conditions are recognizable.

The recommended procedure is to establish a particular temperature recording station which has a demonstrable correlation with the magnitude of the drift. In a "convective environment" this could simply be the "environment temperature."

The temperature of the selected station should be recorded continuously during any measurement process to which the index is to be applied. If the temperature shows a significant change of conditions, the index is null and void for that process, and a reevaluation should be accomplished, or the conditions corrected to those for which the index applies.

In addition to continuous monitoring of environmental conditions, it is recommended that efforts be made to establish that the process is properly soaked out. This may be done by checking the temperature of all elements before and after the execution of the measurements.

#### APPENDIX B: DRIFT-CHECK PROCEDURE

The following is the recommended procedure for the conduct of a drift check for a process in which the proposed monitoring method is based on the measurement of environment temperatures.

##### A. Equipment

The major equipment necessary includes very sensitive displacement transducers and sensitive, drift-free temperature sensors with associated

amplifiers and recorders. A linear variable differential transformer with provision for recorder output has proven quite successful. Also, various resistance-bulb thermometers with recording provision have proven successful as temperature monitoring devices.

The required sensitivity of the displacement transducers used may be adjusted according to the rated accuracy of the measurement system.

#### B. Equipment Testing

The temperature measuring and recording apparatus should be thoroughly tested for accuracy of calibration, response and drift. The availability of sensitivities of at least  $0.1^{\circ}\text{F}$  is desirable. Time constants of sensing elements of about 30 sec are recommended.

Before the displacement transducers and associated apparatus are used they should be calibrated and checked for drift in the environment. An "electronics drift check" should be performed by blocking the transducer and observing the output over a period of time at least as long as the duration of the drift test to be performed. "Blocking" a transducer involves making a transducer effectively indicate on its own frame, base, or cartridge.

#### C. Preparation of the System for Test

An essential feature of the drift check is that conditions during the check must duplicate the "normal" conditions for the process as closely as possible. Therefore, before the check is started, "normal" conditions must be determined. The actual step-by-step procedure followed in the subject process must be followed in the same sequence and with the same timing in the drift check. This is especially important in terms of the actions of any human operators in mastering and all preliminary setup steps.

With as little deviation from normal procedure as possible, the displacement transducers should be introduced between the part (or master, depending on the type of drift check) and the rest of the C-frame such that it measures relatively displacement along the line of action of the subject measurement process.

The temperature sensing pickup must be placed to measure a temperature which is correlatable with the drift. Some trial and error may be necessary. In the extreme case, temperature pickups may have to be placed to measure the temperature of all of the active elements of the measurement loop.

D. Representative Time Period For a Drift Check

Once set up the drift check should be allowed to continue as long as possible, with a minimum of deviation from "normal" operating conditions. In situations where a set pattern of activity is observed its duration should be over some period of time during which most events are repeated. When a 7-day work week is observed in the area, and each day is much like any other, a 24-hour duration is recommended. If a 5-day work week is observed, then either a full-week cycle should be used or checks performed during the first and last days of the week.

E. Postcheck Procedure

After the drift test, the displacement transducers and the temperature recording apparatus should be recalibrated.

F. Evaluation of the Drift Check (Drift-Check Report)

Following the drift check, the data should be assessed for the following values.

- (a) Nonperiodic Effects - the effects of the operator tend to disappear with elapsed time. These and similar effects should be described and the portion of this error not compensated by soak out should be included in the TVE .

- (b) Temperature Variation Error (TVE) -

For zero or small mastering cycle time.

$$\text{TVE} = \left[ \begin{array}{l} \text{Maximum excursion of part-comparator and master-} \\ \text{comparator drift curves when curves are aligned for} \\ \text{in-phase temperature conditions over representative} \\ \text{time period.} \end{array} \right] \quad (\text{Part-to-master drift})$$

For significant mastering cycle times.

$$\text{TVE} = \left[ \begin{array}{l} \text{Master-comparator drift or} \\ \text{part-comparator drift, which-} \\ \text{ever is greater, for chosen} \\ \text{mastering cycle time} \end{array} \right] - \left[ \begin{array}{l} \text{Part-to-master} \\ \text{drift error as above} \end{array} \right]$$

Use whichever of above TVE is greater.

A complete report of the drift check findings should include the following:

#### Thermal Drift-Check Reports Outline

Items in parenthesis are suggested as a guide to what might be pertinent under a heading.

1. Description of System
  - a) Identification  
(Mfgs., model, pertinent specifications, and dimensions)
  - b) Component Motions  
(Active elements, lines of action)
  - c) Operations
    - 1) Type of operation

2) Typical workpiece

Sizes

Materials

Minimum tolerances

3) Method of mastering

4) Cycle times

(Operating, mastering)

2. Environment Description

a) Room Features

(Size; solar exposure; exits; wall, floor, ceiling, and other heat sources)

b) System Features

1) Location with respect to "room features"

2) Internal heat sources

(Motors, lamps, electronics)

c) Air Circulation

(Inlet-outlet locations, sizes, numbers, drafts, air volume circulated)

d) Temperature monitoring and control

3. Test Apparatus Description

a) Temperature monitoring

(Identification, response, sensitivity, location)

b) Displacement monitoring

(Identification, response, sensitivity, location)

4. Procedure

a) Stepwise description of testing

5. Results

(Displacement-temperature vs time graphs; maximum displacements and temperature variations; cycle times; causes if known)

6. TVE

7. Recommendations

APPENDIX C: A METHOD FOR DETERMINING FREQUENCY  
RESPONSE OF A MEASUREMENT SYSTEM

Data obtained from step-change experiments performed on the 15 inch Sheffield rotary contour gage gave: (1) an indication that the system was linear with respect to thermal variations, and (2) the set of data correlating temperature variation with drift.

The basic characteristic of a linear system is that the output (in this case, drift) corresponding to any input (temperature variation) is the sum of outputs corresponding to the components of the input.

This characteristic permits the use of the data of Fig. 7 in computing a predicted drift for a variation of temperature in the environment of the Sheffield gage as follows:

Suppose that the temperature variation is recorded as shown in Fig. C-1. This record can be approximated by straight lines over 7-minute increments. This procedure decomposes the temperature variation into a series of components similar to that of Fig. 7.

The drift corresponding to any one of these components can be determined by scaling the data of Fig. 7.

Figure C-2 shows the resulting set of drift components and their sums which is an approximation to the drift caused by the temperature variation of Fig. C-1.

This computation procedure is cumbersome when done by hand but is easily and quickly done by a digital computer. The results shown in Figs. 8 and 10 of the paper were computed using an IBM 7094 digital computer at the Lawrence Radiation Laboratory.

There are two practical considerations to be observed in making computations of this kind:

- (1) It must be possible to make an accurate approximation to the temperature data. For example, in the case described above, a temperature variation consisting of a sine-wave with a period of less than 7 minutes cannot be approximated by the data available.
- (2) Because of the "memory" of the system, the computed drift is in error until a period of time equal to the soak-out time has elapsed. For example, in computing the data for Fig. 8, the drift computed for the first 12 hours of temperature variation record was inaccurate and was omitted.

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<sup>9</sup>This reference was not received until after the completion of this paper. Mr. Grand has covered much of the same material presented here. His approach to the problem is different, however, and the duplications do not justify withdrawal of this paper.

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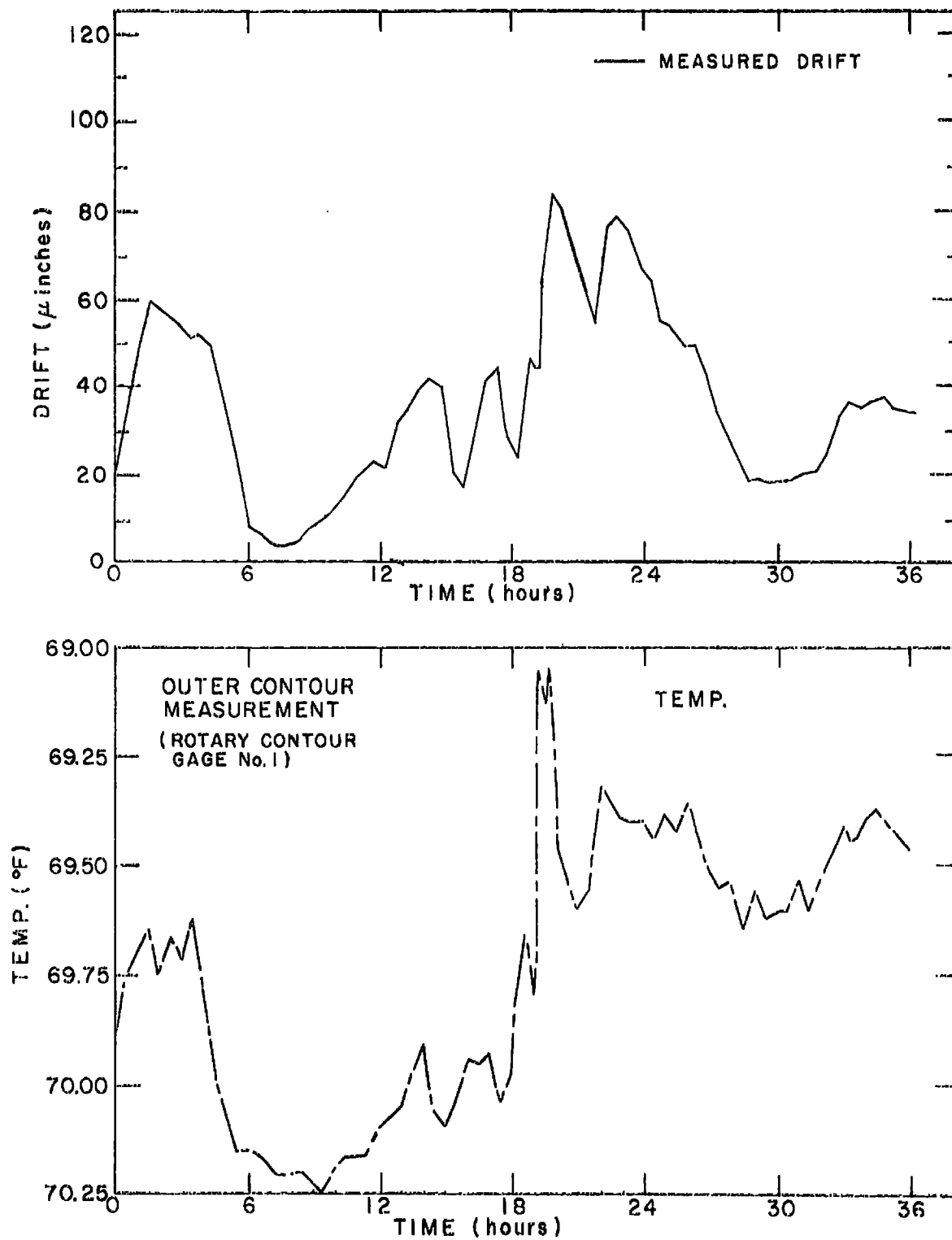


Fig. 1 Thermally induced drift of 15-inch Sheffield rotary contour gage No. 1 with steel part.

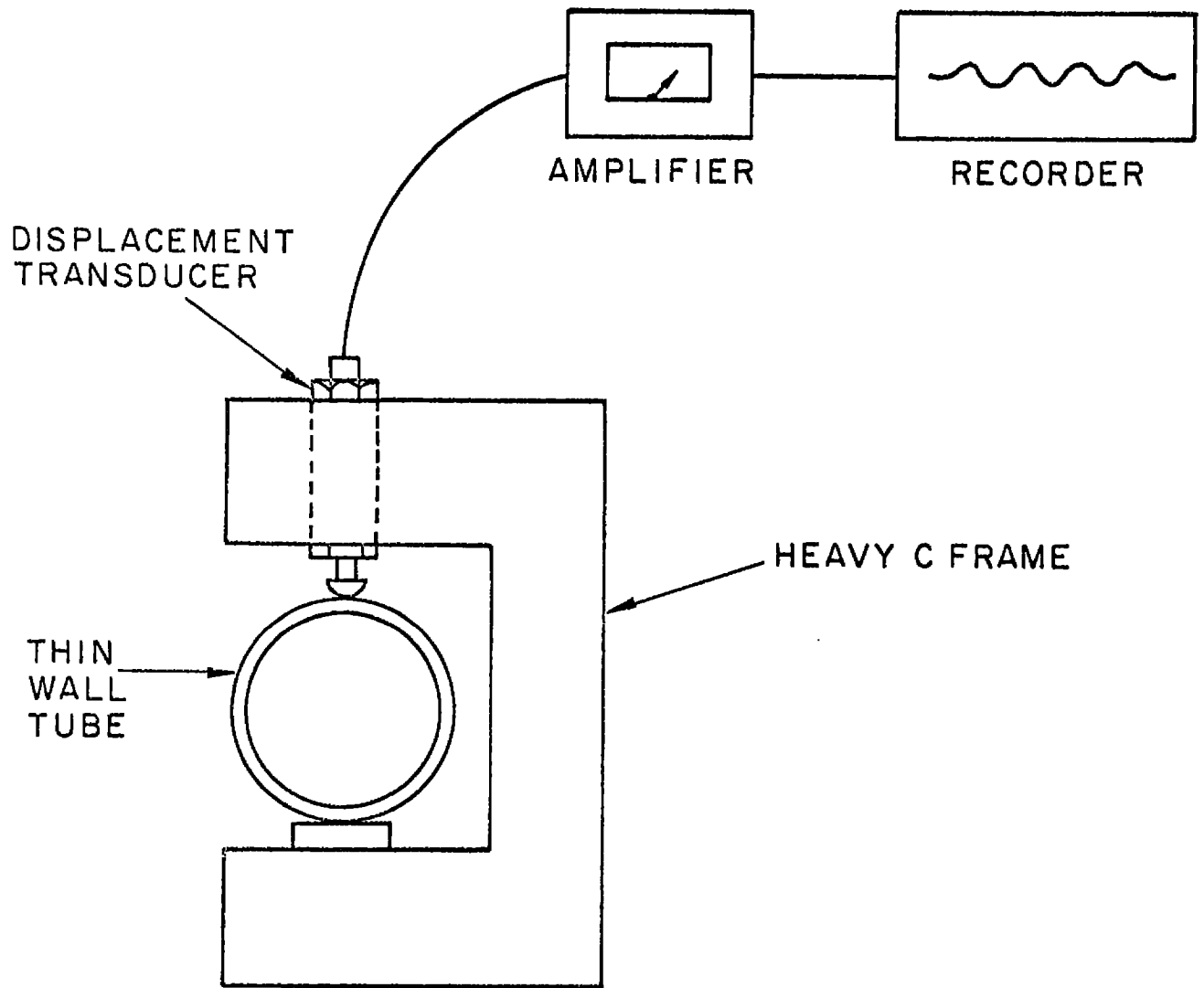
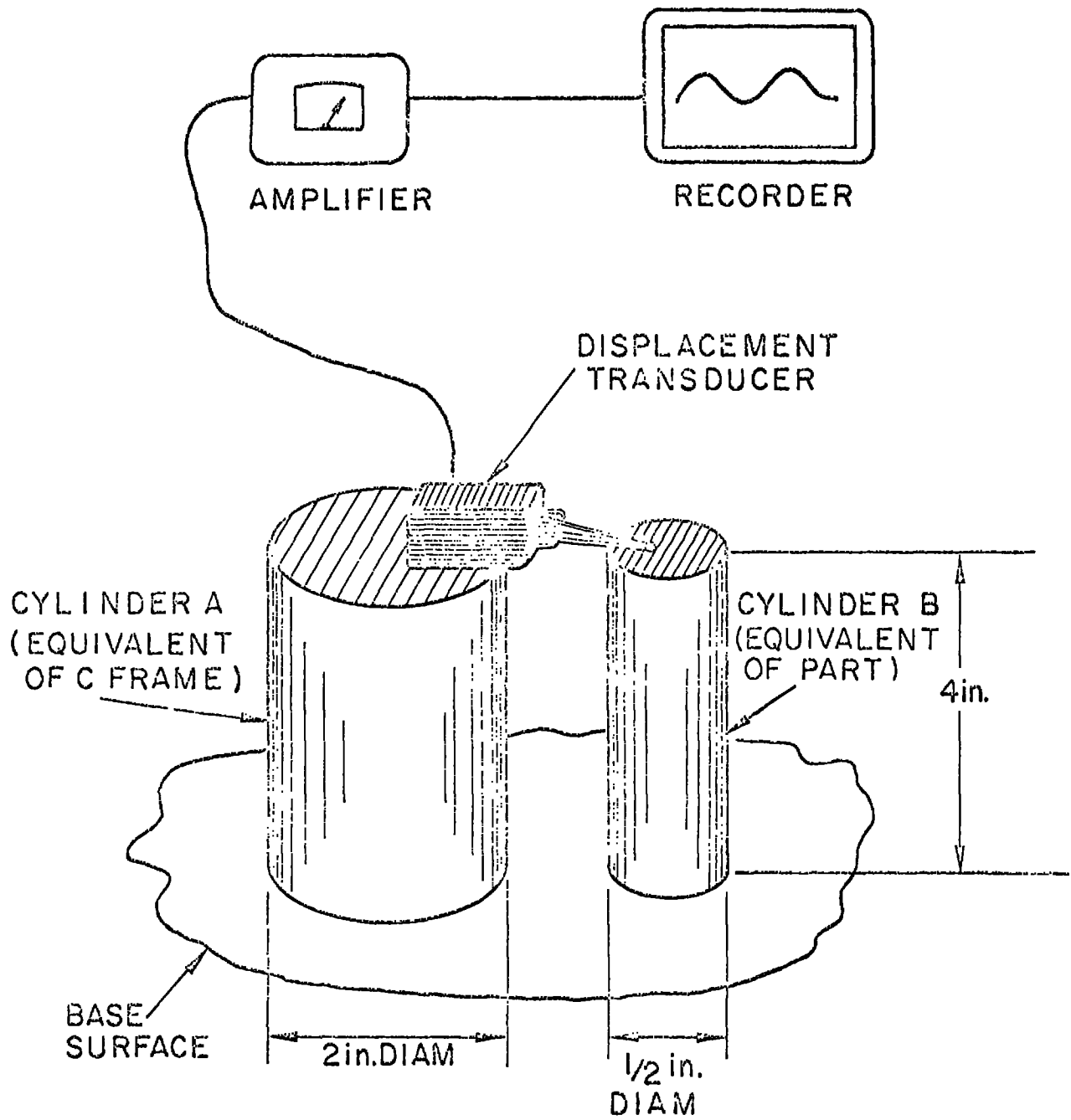


Fig. 2. Schematic of C-frame comparator and part.



MODEL OF "C" FRAME  
COMPARATOR AND PART

Fig. 3. Model of C-frame comparator and part.

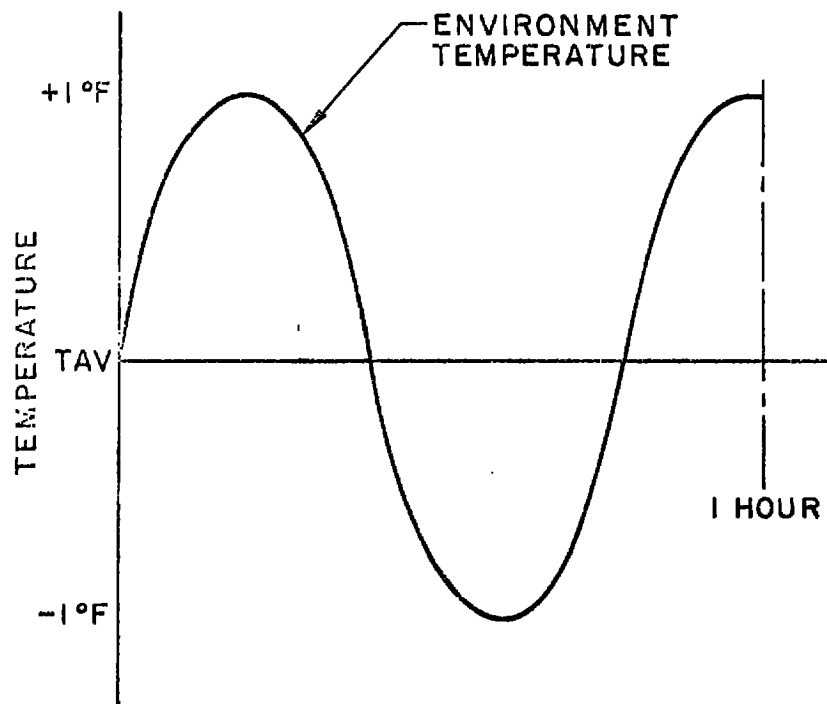
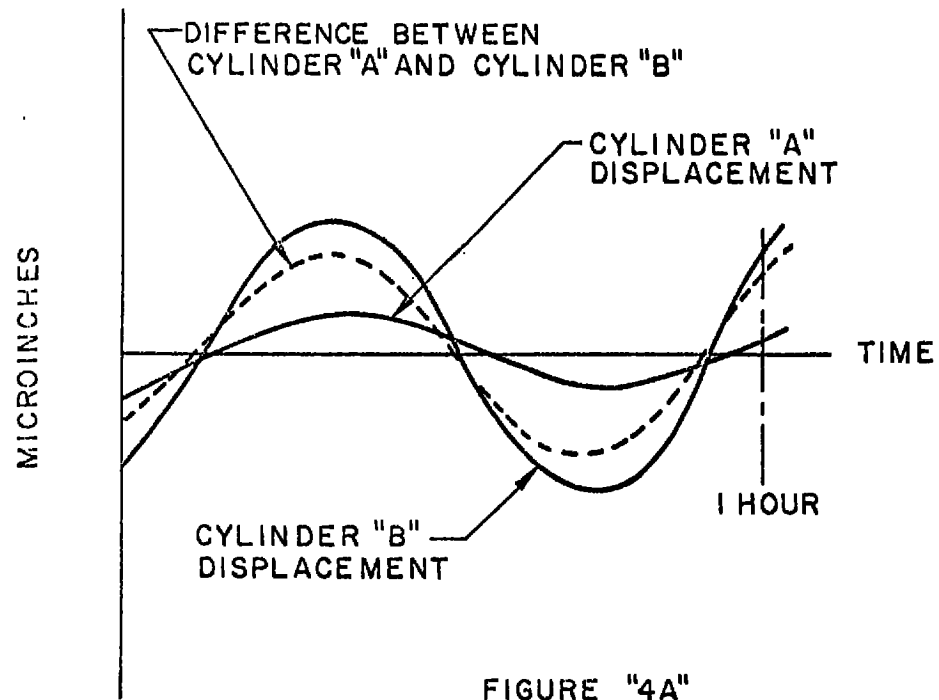


Fig. 4. Individual and net drift thermal response of two-element model shown in Fig. 3.

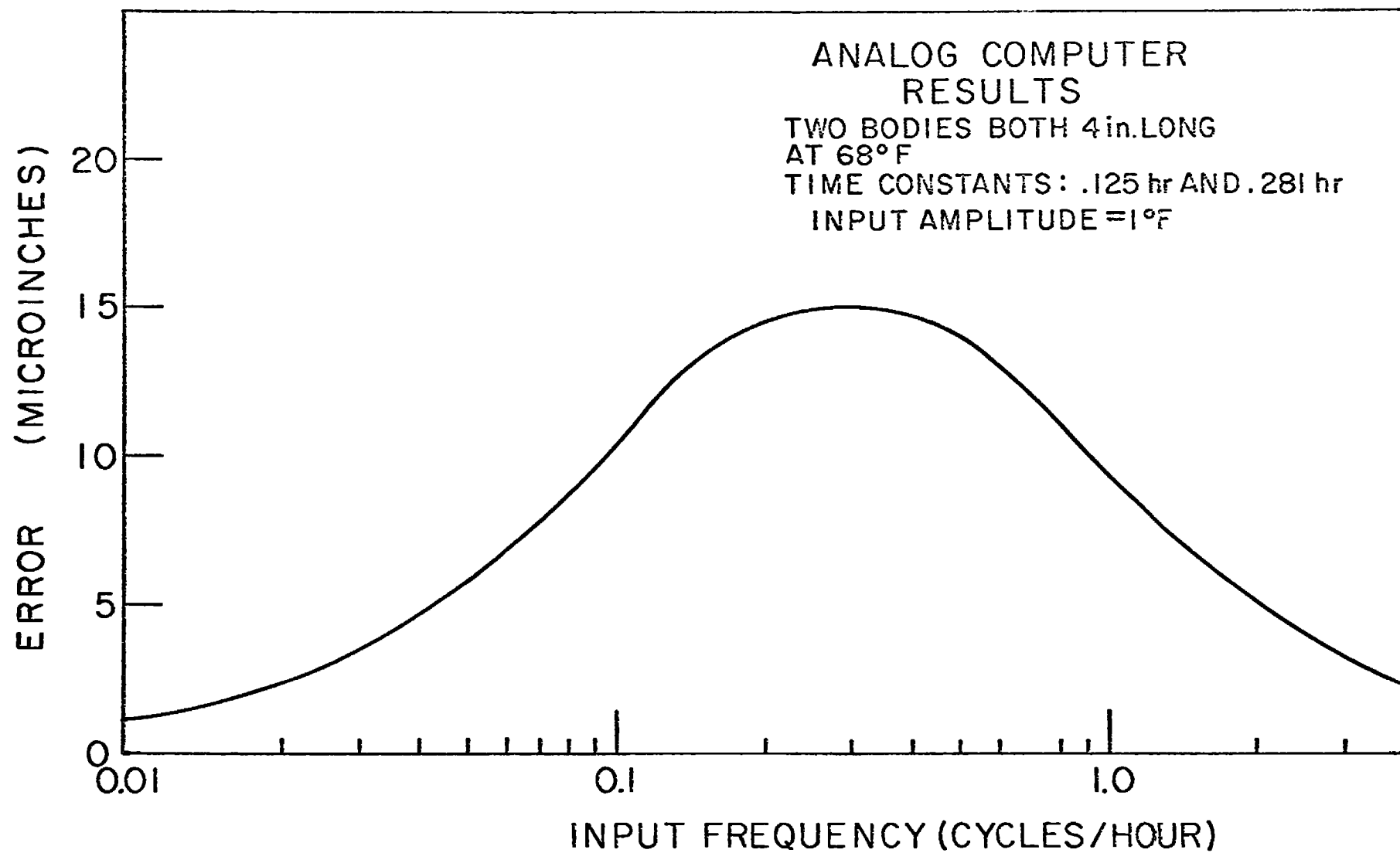
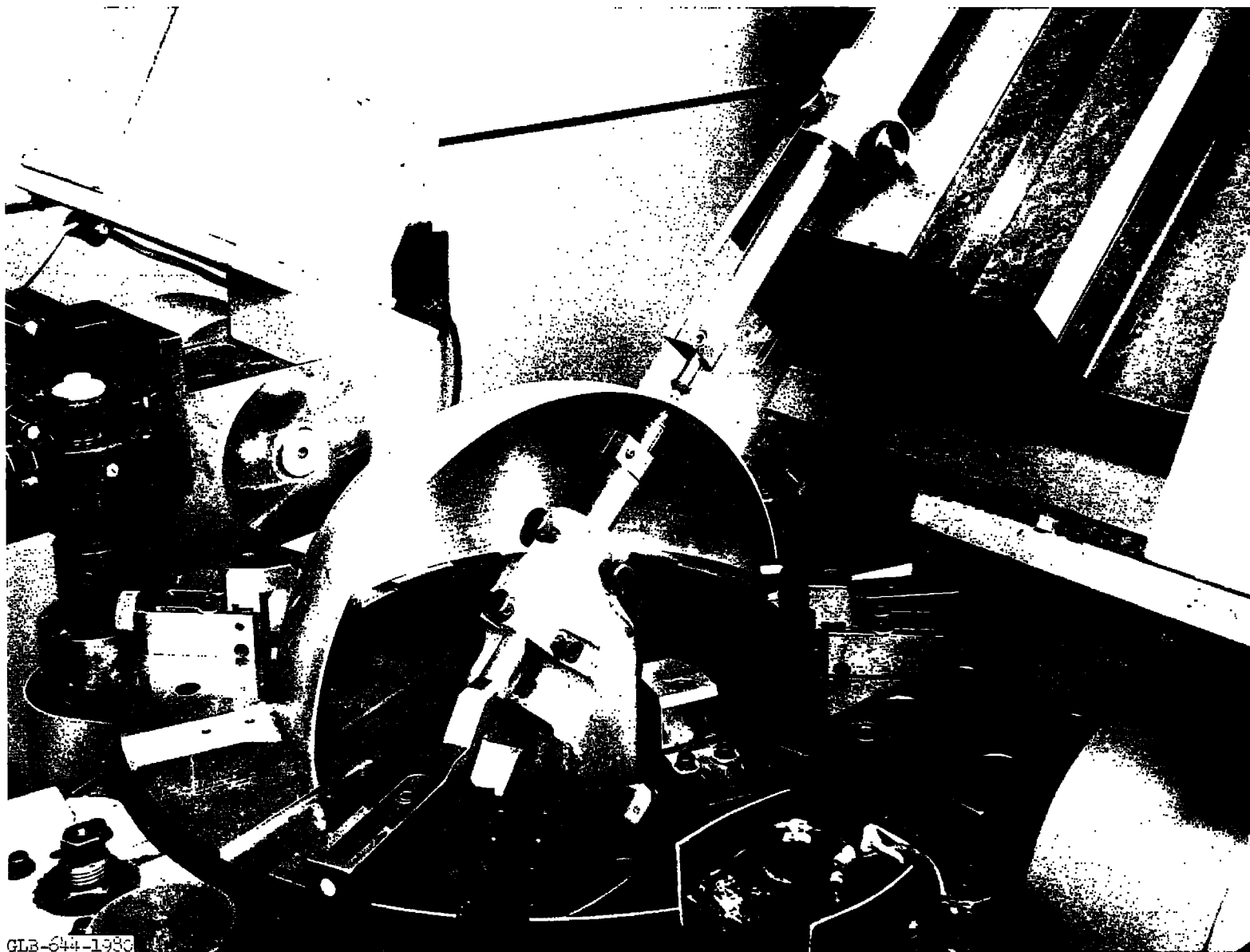


Fig. 5. Computed frequency response of C-frame comparator and part.





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Fig. 6. Partial view of 15-inch Sheffield rotary gage with thin-walled steel part.

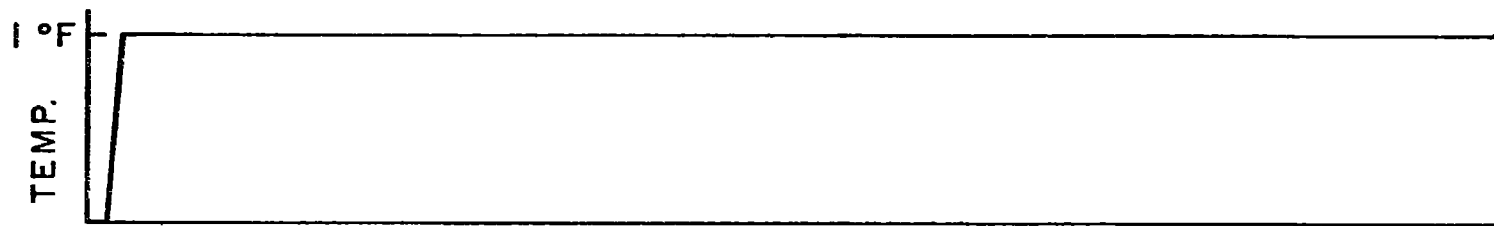
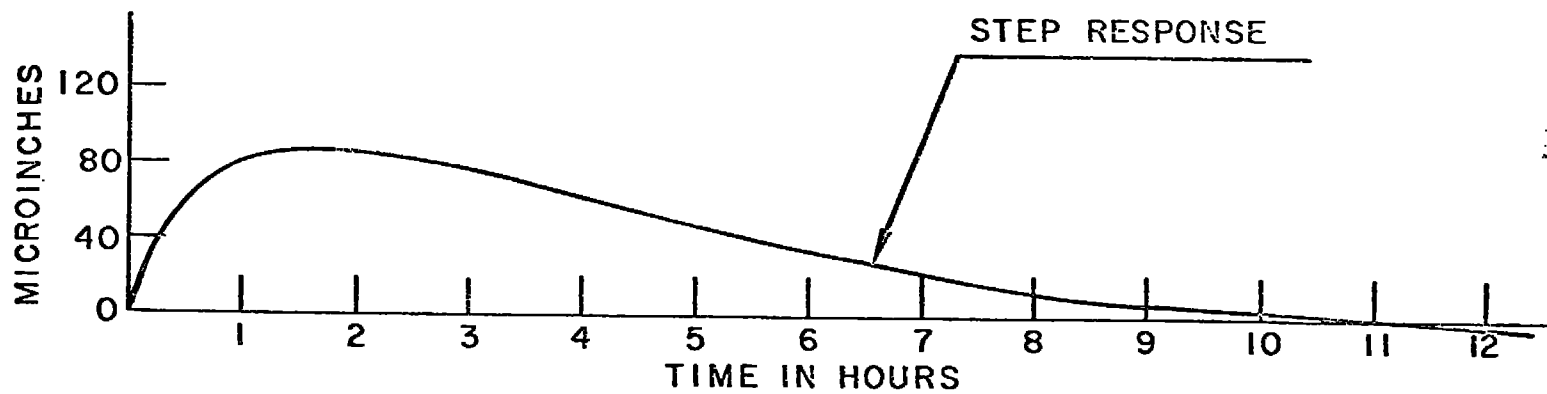


Fig. 7. Step input response on 15-inch Sheffield rotary contour gage No. 2.

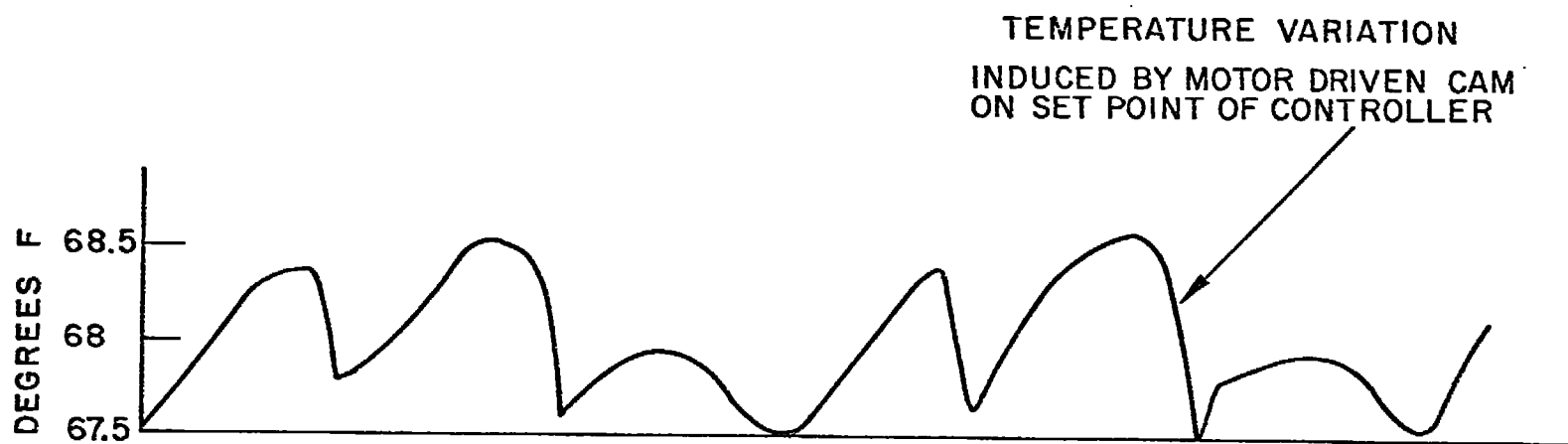
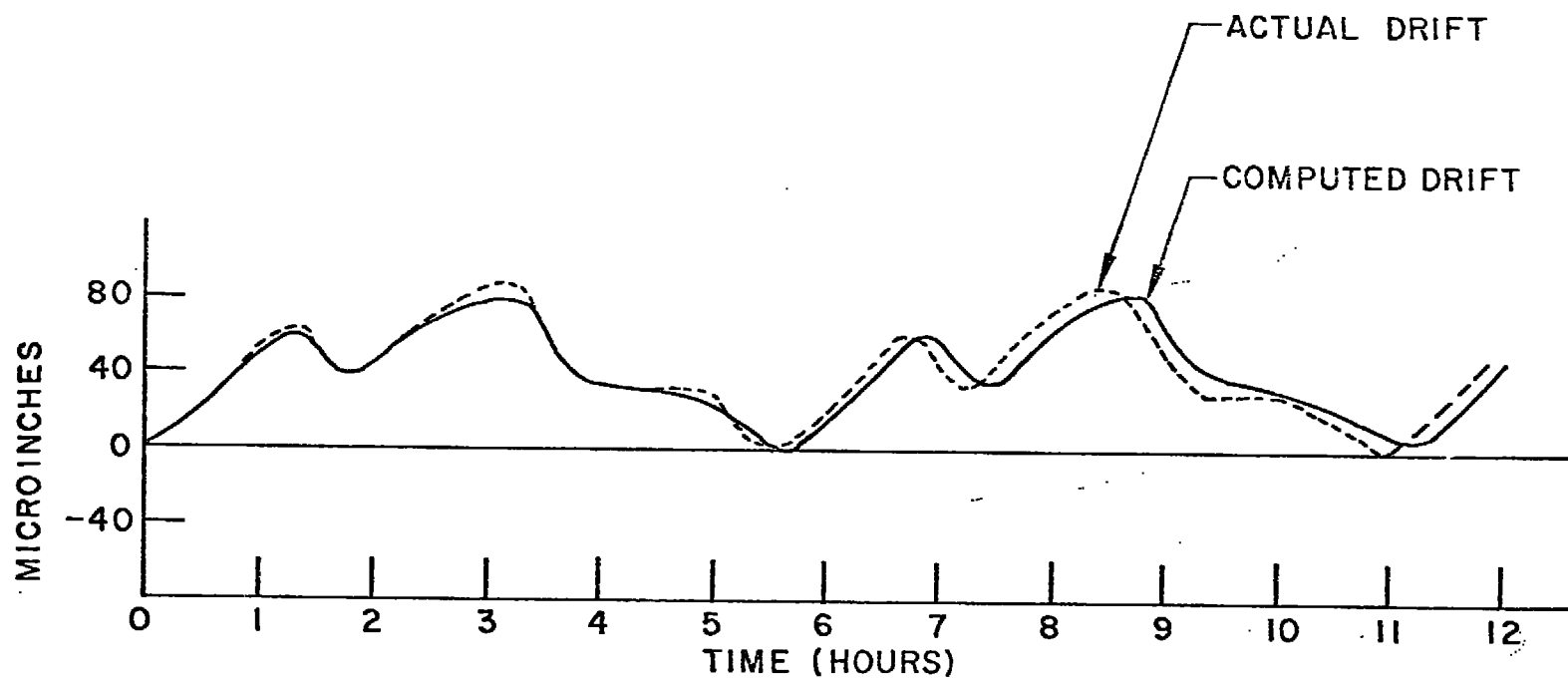


Fig. 8. Forced drift check on 15-inch Sheffield rotary gage No. 2.

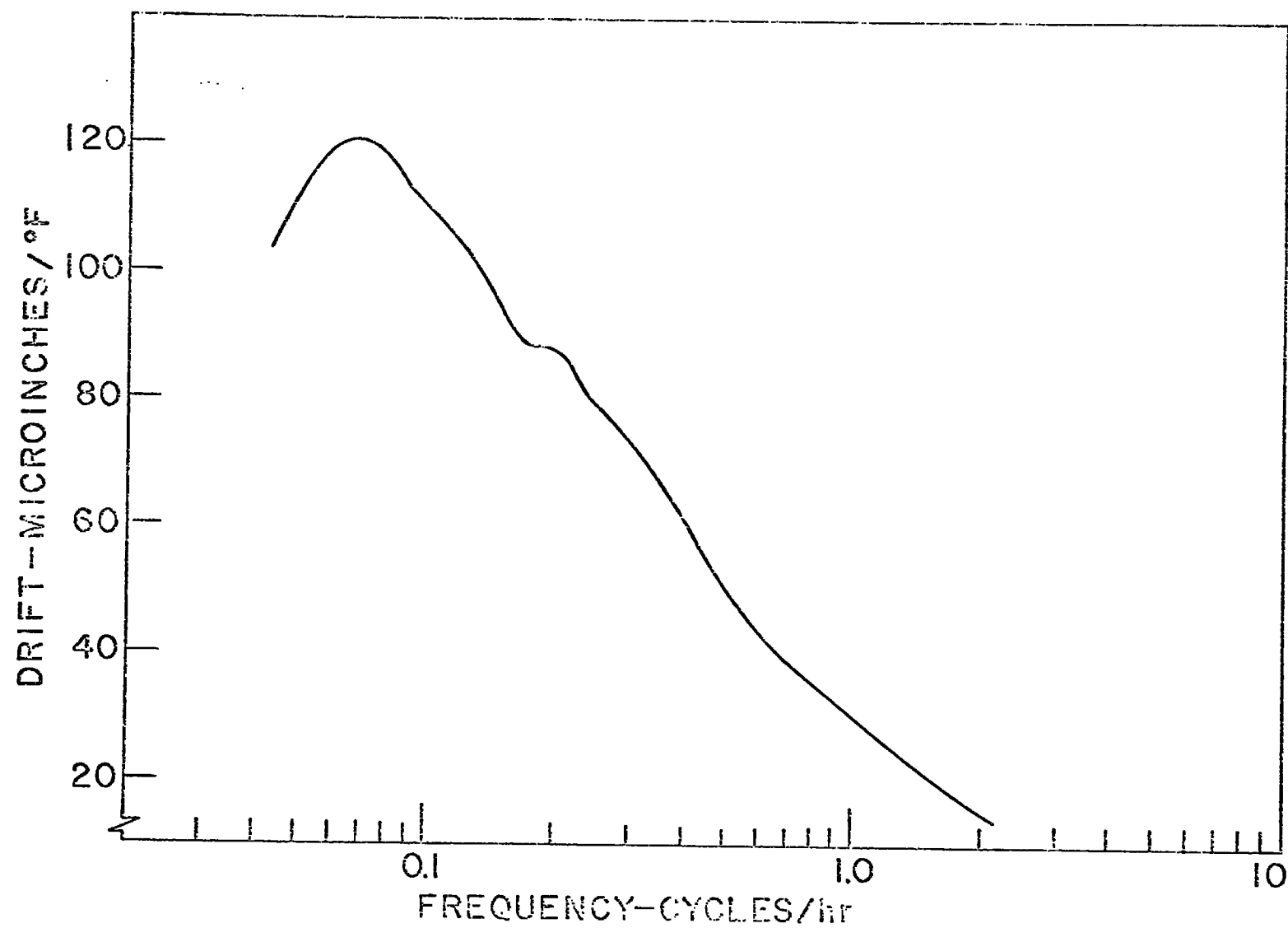


Fig. 9. Computed frequency response of Sheffield rotary contour gage No. 2.

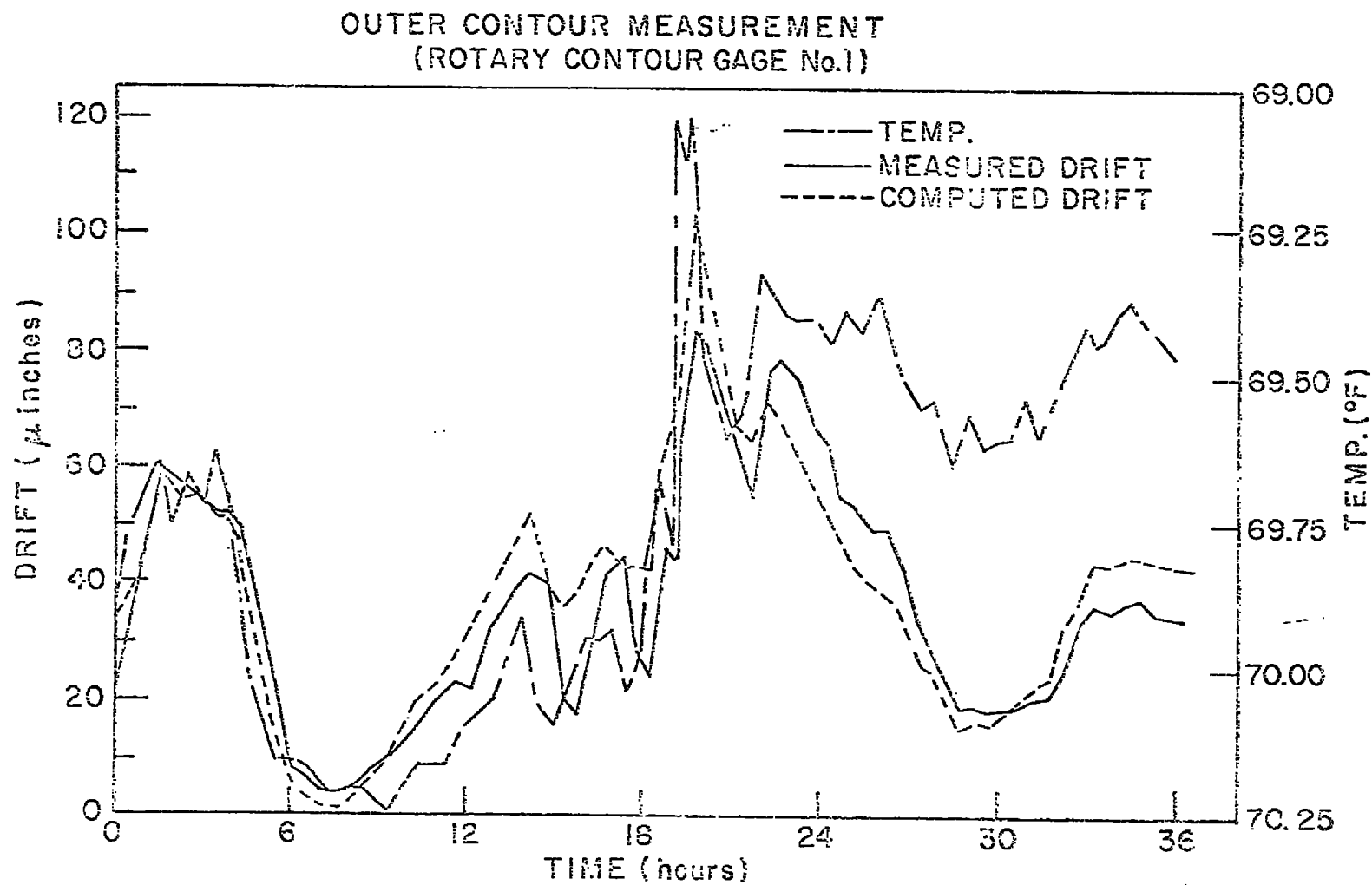


Fig. 10. Computed and actual drift of Sheffield rotary contour gage No. 1 with steel part.

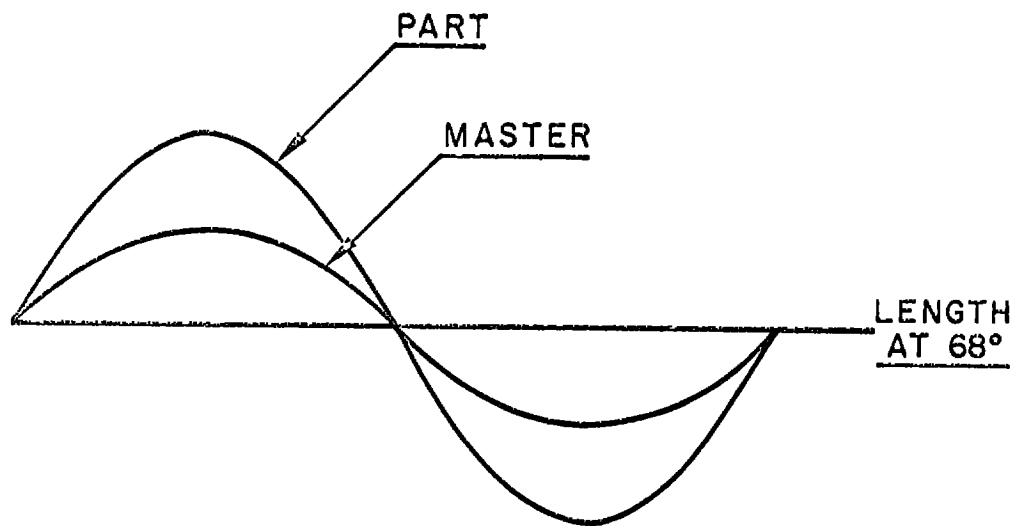
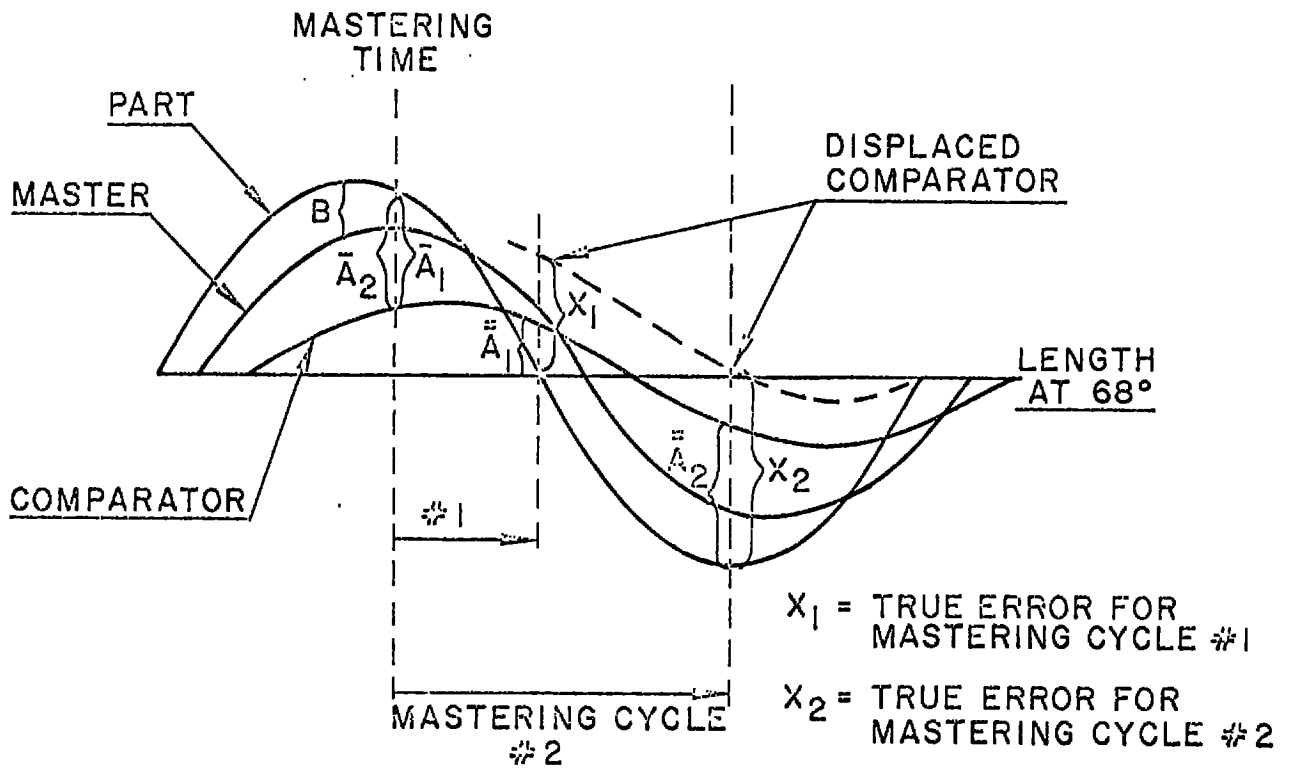


Fig. 11. Part-to-master drift using in-phase, sinusoidal curves.

B = PART-TO-MASTER MAXIMUM ERROR

A = MAXIMUM COMPARATOR DRIFT  
FROM PART OR MASTER  
DURING MASTERING CYCLE

A - B = APPROXIMATE TVE FOR  
MASTERING CYCLE SHOWN



$(\bar{A}_1 + \bar{A}_1) - B \sim X_1$   
 $(\bar{A}_2 + \bar{A}_2) - B \sim X_2$

FOR SINUSOIDAL CURVES SHOWN,  
 CALCULATED APPROXIMATE ERROR (A-B)  
 IS SOMEWHAT LESS THAN TRUE ERROR (X)

Fig. 12. Drift error for two mastering cycle times.

$B$  = PART-TO-MASTER MAXIMUM ERROR

$\bar{A} + \bar{\bar{A}}$  = MAXIMUM COMPARATOR DRIFT  
FROM MASTER

$X_1$  = TRUE ERROR IF PART IS  
INDICATED AT MASTERING TIME

$X_2$  = TRUE ERROR IF PART IS INDICATED  
AT END OF MASTERING CYCLE TIME

$A - B < B$  FOR EXAMPLE SHOWN REGARDLESS  
OF MASTERING CYCLE TIME

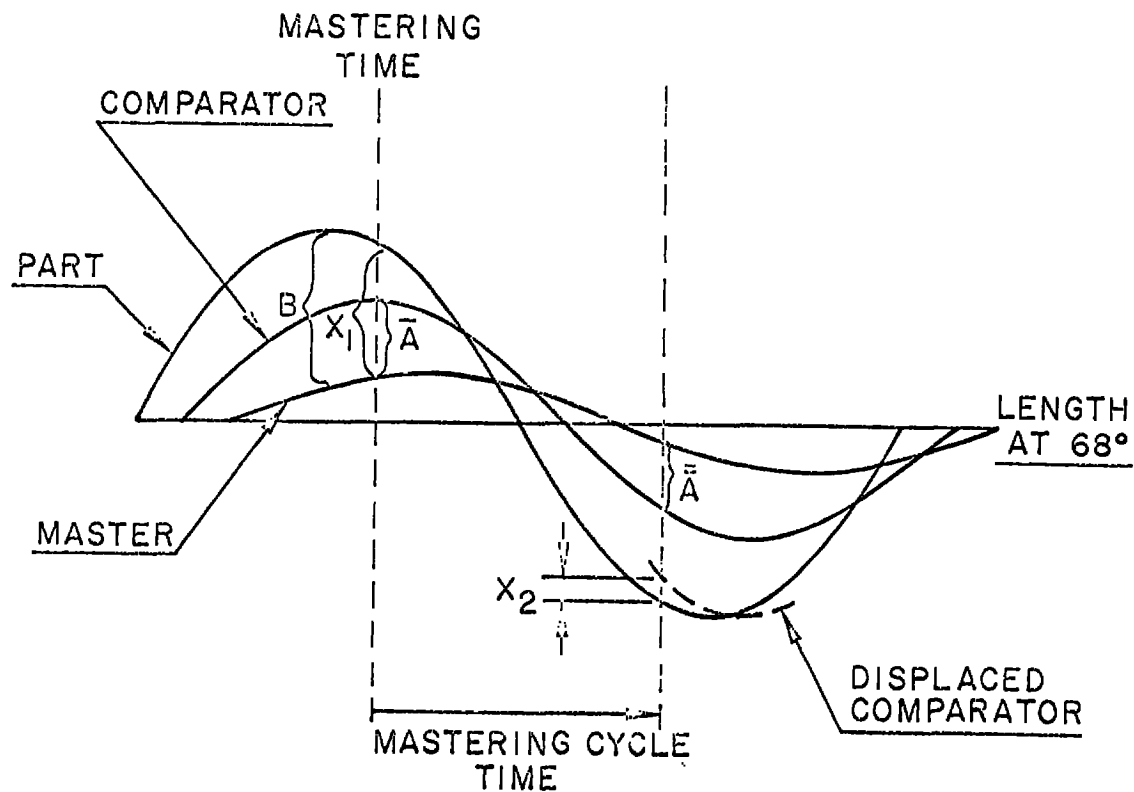


Fig. 13. Drift error when comparator drift is between master and part.



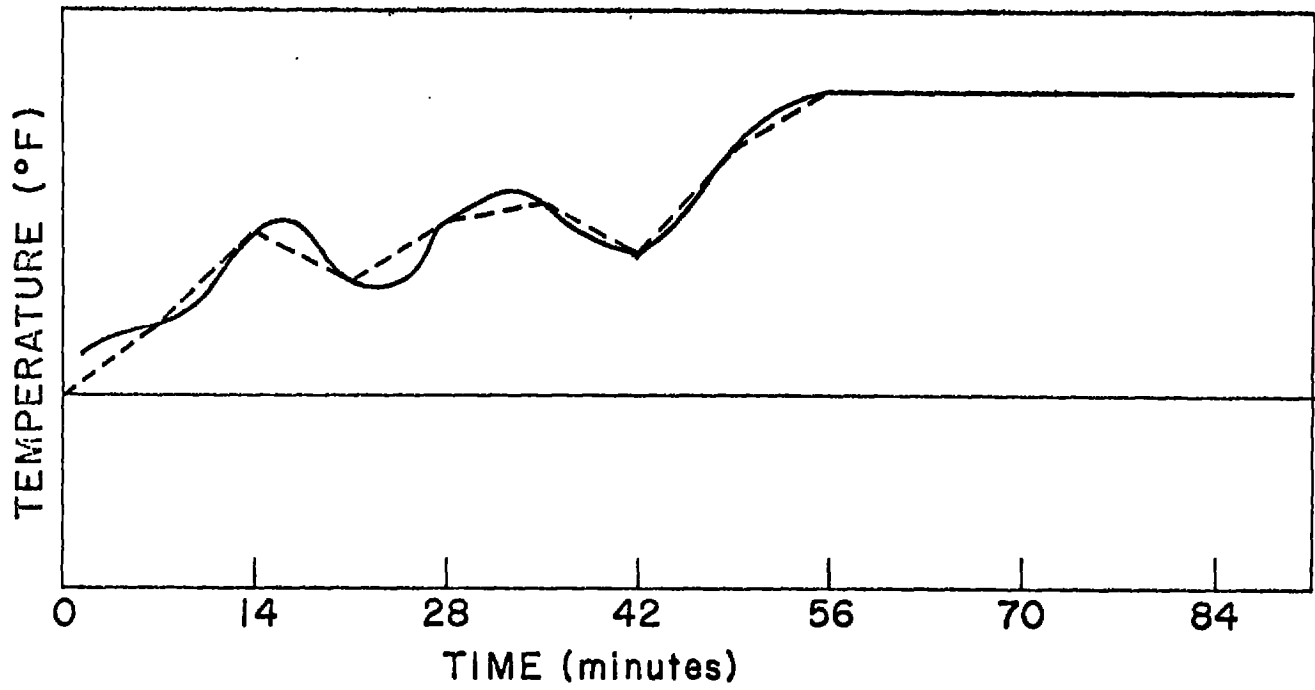


Fig. C-1. Temperature variation and approximation.

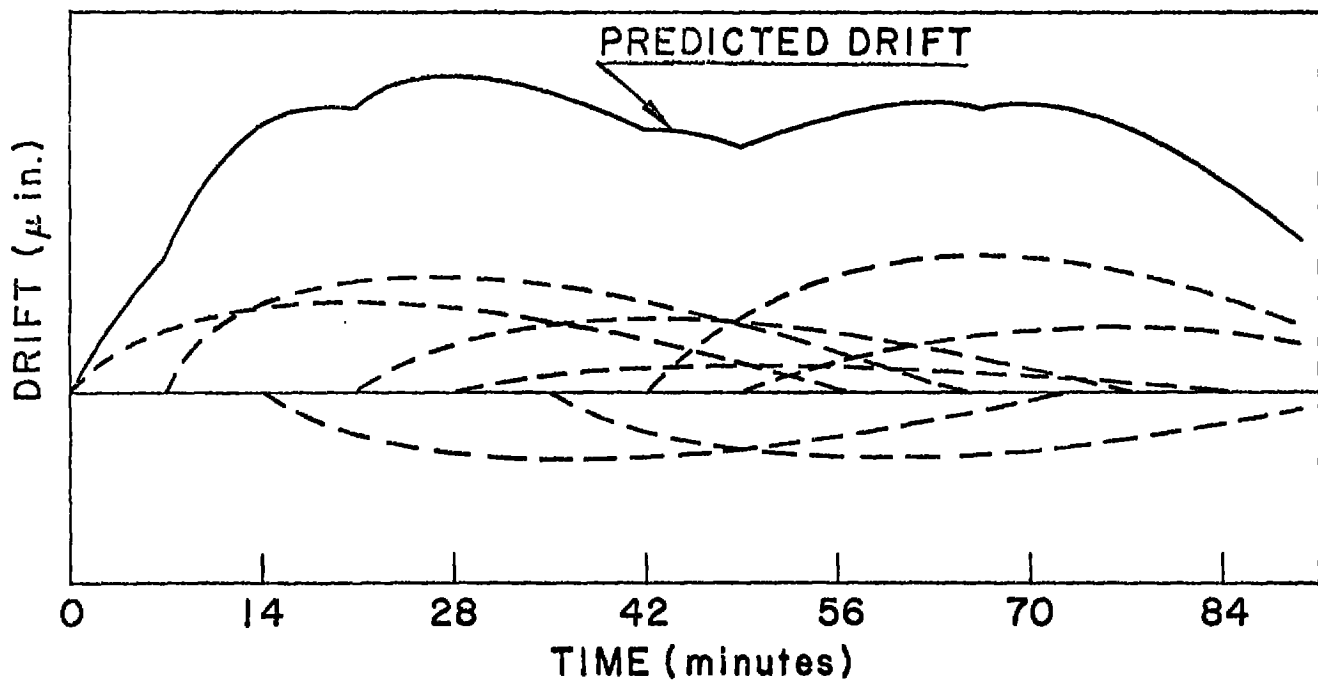


Fig. C-2. Graphical addition of drift components.

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